

A scientist and a poet

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Vladimir Zakharov was a man of a strong passion and grand intellect, who was equally and deservedly proud of both his scientific achievements and his poetry.

The following short poem called *Theoretical Physicist* was composed by Vladimir Zakharov, the great mathematical physicist, who left us on 20 August this year at the age of 84.

*Like a minuscule grey-coloured cricket,
A wee man by a blackboard is clicking.
That's his way to invite for a mosey
Across worlds that not many find cosy.
But as soon as it comes to departure,
The attendance appears rather patchy:
Only few still persist at note-taking!
Tree of knowledge, thy fruit is bliss-making...
Keep your hands off our light entertainment,
Do not tempt us with crumbs of attainment,
Do not teach us the right aspirations,
Do not tease us with serving the nation.*

[Translated from the Russian by A. Shafarenko¹]

It is no accident that mathematics and poetry rhyme. Each, using its own language, seeks to strip away the superfluous, escape traditional views and logic, and, in doing so, reveal universal truths. Mathematics and poetry demand simultaneously great passion and clear-minded tireless attention to detail, and Zakharov was equipped with both.

He has contributed in so many areas: from the Zakharov equations of plasma physics and work on self-focusing singularities and wave collapse through to his papers on the integrability of nonlinear partial differential equations, soliton theory and exact solutions for wave turbulence. He seemed to have the knack of being the first at getting to all the good problems. Zakharov had a passion for building theoretical methods that describe, in the same mathematical language, physical phenomena that are conventionally considered as non-related. He was a genius, brilliant and intuitive.

There were four landmark developments in the nonlinear sciences in the 1960s: wave turbulence, integrable systems, collapses and chaos theory. Zakharov played a central part in breakthroughs in the first three of these.



Vladimir Zakharov

Turbulence is at present understood as a far-from-equilibrium state of any system with many degrees of freedom. The problem of turbulence permeates most fields in science and engineering, from fluid mechanics and plasma to optics and atomic physics. Zakharov probably did more than anyone to establish this unifying view by bringing the vast domain of far-from-equilibrium wave systems to be considered along with traditional fluid turbulence. The kinetic equation for wave turbulence theory was derived by Klaus Hasselmann under statistical assumptions, assumptions then shown to be unnecessary by Philip Saffman, David Benney and Alan Newell, and Zakharov's main achievement in wave turbulence theory was to identify the statistically steady states corresponding to finite flux spectra. This nontrivial step was as monumental as Kolmogorov's in fluid turbulence: both derived exact mathematical results that have guided us for over half a century. The turbulence of a sea of weakly interacting random waves is described by a closed kinetic equation, and Zakharov found the amazing exact solutions. They are relevant to various fields of physics dealing with wave turbulence: plasmas, optics, acoustics, magnetic waves, Alfvén waves, Rossby waves and more. Because of their connection with Kolmogorov's work in turbulence, these exact

solutions are now known as the Kolmogorov–Zakharov solutions (details and original references can be found in ref. 2). For this work and particularly for the discovery of inverse flux solutions, he and Robert Kraichnan were awarded the Dirac Medal in 2003.

In a seminal paper³, Zakharov and Aleksei Shabat demonstrated the integrability of the nonlinear Schrödinger equation (NLSE), a remarkably generic and canonical model of the nonlinear sciences, with a vast number of applications ranging from ocean waves, plasmas, acoustics, hydrodynamics, and bio-physics to Bose–Einstein condensates and nonlinear photonics, including self-focusing, dynamics of spatial lightwave packets, fibre-optics and high-speed optical communications, to name just a few. This generality results from the fact that the NLSE describes nonlinear wave envelopes of weakly nonlinear, strongly dispersive systems. (We note that the word ‘weakly’ here has a relative meaning; high-powered light beams can be mathematically treated as weakly nonlinear.)

That breakthrough NLSE paper³ inspired much research into the rich mathematical properties of the inverse scattering transform (IST), from the solution of the nonlinear partial differential equation itself to the scattering data and any associated bound states. The IST method was used first by Clifford Gardner, John Greene, Martin Kruskal and Robert Miura in their seminal work on the integrability of the Korteweg–de Vries equation. The IST method is one of the greatest achievements of mathematical physics in the twentieth century. In plain terms, the IST method can be considered to be an analogue or generalization of the conventional Fourier transform to the nonlinear Fourier transform. IST presents solutions of complex nonlinear evolution equations in the basis of noninteracting modes called a nonlinear spectrum, that includes a continuous part (similar to the standard spectral Fourier harmonics) and discrete eigenvalues, that correspond to soliton modes – localized nonlinear structures. After this pioneering work³, the number of integrable systems exploded – from a tiny number, which literally could be counted on one hand, including the two-body problem and certain spinning tops, to infinite sets of partial differential equations. Soliton

factories sprang up all over the world, bringing soliton theory into various scientific and engineering applications. Moreover, the nonlinear Fourier transform can be applied to dissipative systems, such as lasers, to offer a description of dissipative solitons in the space of fewer parameters than in a traditional Fourier frequency presentation.

Localization of the distributed field energy in time or space by nonlinearity is a general physical phenomenon observed in various areas of science as diverse as hydrodynamics, plasmas, nonlinear optics, molecular biology, field theory and astrophysics. The many ramifications of the soliton concept, from pure mathematical theory to engineering technologies implemented in practical devices, embody the importance and impact of the soliton theory. The prevalence of solitons is largely due to a relatively small number of versatile nonlinear equations governing a wide range of physical and biological systems, such as the NLSE. Zakharov and his school pioneered numerous important results in soliton theory, ranging from soliton stability to soliton gas concepts^{4,5}.

Zakharov also made pioneering contributions in another core area of nonlinear science – the notion that certain nonlinear wave equations in dimensions greater than one can have localized solutions that become infinite, or ‘collapsed’, in finite time⁶. Indeed, the balance between dispersion/diffraction and nonlinearity can be stable, leading to solitons, or unstable⁵, leading to wave collapse, with the self-focusing of light being the well known example of such explosive wave dynamics. Collapse can be treated as the reverse process to the explosion starting from a singularity (the Big Bang being the most famous example of such highly nonlinear dynamics). The study of collapses and how they occur may provide insights into the challenge of understanding dark energy – this mysterious universal substance that lies between galaxies of ordinary matter, whose pressure increases as its density decreases, leading to an ever-accelerating expansion of the Universe.

Each of these remarkable general nonlinear science breakthroughs has had and will continue to have a direct and profound impact on nonlinear photonics. The NLSE alone is a very important underlying mathematical model in optics, with applications in all-optical signal transmission and processing, pulse compression, supercontinuum generation, optical pulse shaping, frequency conversion, and the design and operation of nonlinear photonic devices, to mention a few. The multi-soliton

solutions found in ref. 3 are directly relevant to optical pulse compression, and the initial stage of supercontinuum generation. The IST method allows us to follow the modulation instability of light waves to the nonlinear stage. Although modern coherent detection communication systems do not directly use solitons for transmission of information, the NLSE, within certain limits, is still the master model for various fibre optics communication channels. Therefore, IST-based optical signal processing might unlock new techniques for the analysis of conventional signals and enable us to develop new modulation formats appropriate for nonlinear communications channels. This work is still very much in progress.

In his relatively less well known paper of 1970, using the general Hamiltonian approach, Zakharov and his student⁷ derived equations for wave propagation with two polarizations in nonlinear dispersive media. These results were used later by another student of Zakharov to introduce a new integrable model – the Manakov equations, which are widely used nowadays in the design of modern optical communication systems. The Hamiltonian methods developed by Zakharov and his school⁸ offer a general approach to soliton stability⁵ and the notion of quasi-solitons, which emit radiation due to resonances with the linear dispersive waves (optical soliton Cherenkov radiation⁹). Optical wave turbulence in fibre optics and lasers is another area where ideas pioneered by Zakharov and his school create a platform to understand physical nonlinear systems (and corresponding practical engineering devices) with many degrees of freedom far from equilibrium, that is highly relevant to high-power fibre lasers. Zakharov and his students developed the nonlinear theory of superfluorescence, demonstrating within the framework of the Maxwell–Bloch model that in the limit of neglected relaxation and inhomogeneous broadening, the pulse dynamics of superfluorescence is self-similar¹⁰.

Zakharov was an honourable man of enormous integrity that he demonstrated throughout his life. He was as generous in giving credit as he was sharp in dismissing nonsense. He would not hesitate to say: “You are moving science sideways”. He stood up to bullies, demagogues and purveyors of pseudoscience nonsense, and spoke truth to power, which often cost him great personal freedoms. For example, in 1968, he signed a letter calling for the release of Soviet prisoners of conscience, which resulted in him being prohibited from travelling abroad for twenty years. Another example from Soviet times – he insisted

that the Lenin Prize committee include Alexey Shabat, together with him and Ludvig Faddeev, in the nomination for the prize for IST, and declined to be nominated without his co-authors. In post-Soviet times, especially in the 1990s, he fought, together with Vladimir Fortov and others, for Russian science, the Russian Academy of Sciences (RAS), and in particular for the Landau Institute, where he was a director until 2003. Together with Eduard Kruglyakov and Evgeny Alexandrov, he did much in the fight against pseudoscience, sometimes with high personal risks. In 2013, he was one of the organizers of the informal club of members of the Russian Academy of Sciences (‘Ist of July’) that opposed the reform of the Academy. His speeches in defence of science at general meetings of the Russian Academy of Sciences were always accompanied by applause. At the RAS he was a tuning fork of conscience.

Zakharov’s uncompromising passion for science is as famous as his brilliant scientific results. He was a man who excelled at both science and poetry. He had a phenomenal memory for science, poetry and history. He treasured science, the arts and the companionship of close friends.

The domain of knowledge is continuously and swiftly expanding, making it impossible nowadays to read all the papers published even in the relatively narrow areas in which we work. Soliton theory and nonlinear science are examples of the beautiful connections between very divergent fields of science through similar underlying mathematical models, just as sometimes poetry can amazingly link completely unrelated objects. Zakharov, with his vision of building a universal nonlinear wave theory using the Hamiltonian approach, built a platform that we can use across such different fields of science. He was a polymath, reminding us of the giants of the Renaissance. It is impossible to list all of his important contributions to nonlinear optics within the scope of this short text.

What is the difference between a genius and a highly talented person? It is next to impossible to sharply define it, yet we know when we see it. This is not about Nobel prizes and other prestigious awards, this is not about citations and other formal things, we just know when we see it.

*“Raise high the roof beam, carpenters.
Like Ares comes the bridegroom, taller
far than a tall man” [Sappho]*

Our feeling was that we saw it in Vladimir Zakharov. Genius sees the world

differently and this vision changes the realm, creating own worlds. Vladimir Zakharov left our world, but his worlds in science and poetry will stay with us and will live their own lives.

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