DIFFUSION OF A PLASMA IN A MAGNETIC FIELD DUE TO COLLISIONS

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A microwave method has been used to study diffusion in the afterglow of a helium plasma. The investigation has been carried out for experimental parameters such that the diffusion across the magnetic field is due to electron-ion collisions. The measured diffusion coefficients in magnetic fields ranging from 700-800 Oe are in agreement with those computed on the basis of classical Coulomb collision theory. At magnetic fields ranging from 1000 to 5000 Oe there is an appreciable deviation from theory, in which case the diffusion coefficient is proportional to 1/H. We have obtained an asymptotic solution of the diffusion equation describing the density decay in the afterglow of a completely ionized plasma in the axially symmetric case.

Introduction

In recent years there has been a great deal of interest in the nature of the diffusion of a fully ionized plasma in a magnetic field. Experiments in this connection have been carried out with a thermal cesium plasma [1, 2]. However, the coefficients describing diffusion in a fully ionized plasma can be measured with plasmas in which the degree of ionization is small, provided the conditions are such that the Coulomb collisions predominate in the diffusion flux across the magnetic field. These conditions can be produced in an afterglow because in an electron gas at low temperatures the cross section for electron-ion collisions is many orders of magnitude greater than the cross section for electron-neutral atom collisions [3, 4].

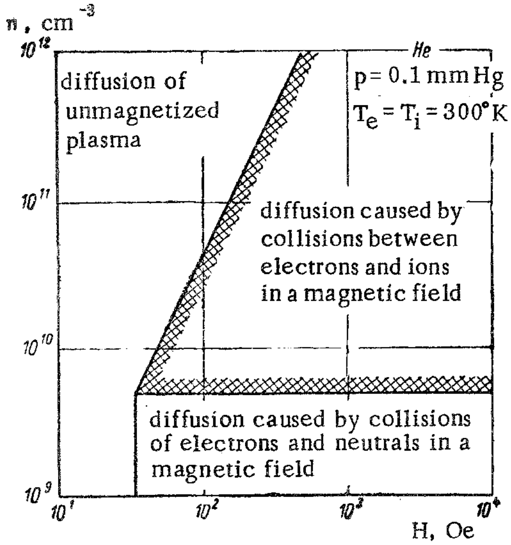

Fig. 1. Diagram of diffusion modes in a three-component plasma.

The region of diffusion due to Coulomb collisions in a three-component plasma as a function of electron density $\underline{n}$ and magnetic field is shown in Fig. 1. We consider here the typical case of decay of a helium plasma produced by a pulsed microwave discharge. The lower value of the density is that for which the electron-ion collision frequency is equal to the electron-neutral collision frequency. The upper limit is the border between the region of Coulomb diffusion and the region of diffusion due to ion-neutral collisions. This transition occurs when the coefficient of Coulomb diffusion across the magnetic field $D_{ei}^{\perp}$ increases with increasing electron density to such a great extent that it becomes comparable to the diffusion coefficient in the absence of a magnetic field, $D_{ia}^{\parallel}$. It is evident from Fig. 1 that a rather large region of diffusion corresponding to Coulomb collisions is available for experiments.

Theory of the Experiment

The variation in the density of charged particles in a plasma in which there are no volume processes (ionization, recombination, etc) is described by the usual diffusion equation

$$\frac{\partial n}{\partial t} = \nabla (D \nabla n). \tag{1}$$

The investigation of plasma decay is carried out in a long thin tube ($L \gg R$) in a fixed longitudinal magnetic field. If $\frac{D_{ei}^{\perp}}{R^2} \gg \frac{D_{ia}^{\parallel}}{L^2}$, a condition that is satisfied in the present experiment, the flux along the axis can be neglected and the diffusion equation for cylindrical geometry becomes

$$\frac{\partial n}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(D_{ei}^{\perp} r\frac{\partial n}{\partial r}\right). \tag{2}$$

The coefficient of ambipolar diffusion across a magnetic field due to Coulomb collisions is given by [5, 6]

$$D_{ei}^{\perp}=\alpha n, \tag{3}$$

where

$$\alpha=\frac{8\pi}{3}\left(\frac{ce}{H}\right)^2\left(\frac{m}{kT}\right)^{1/2}\ln\Lambda.$$

Here e and m are the charge and mass of the electron; k is the Boltzmann constant; H is the magnetic field; T is the equilibrium temperature of the plasma (it is assumed that the plasma components reach thermal equilibrium); $\ln\Lambda$ is the Coulomb logarithm, which is defined as follows [6]:

$$\ln\Lambda=\ln\left[\frac{3}{2}\frac{1}{e^3}\left(\frac{k^3T^3}{\pi n}\right)^{1/2}\right]. \tag{4}$$

In the present case the quantity α can be taken to be independent of coordinates and time.

Converting to the dimensionless variables

$$y=\frac{n}{N_0};\ x=\frac{r}{R};\ \tau=\frac{t}{\tau_0};\ \tau_0=\frac{R^2}{\alpha N_0};\ N_0=n|_{r=0,\ t=0}, \tag{5}$$

we write Eq. (2) in the form

$$\frac{\partial y}{\partial\tau}=\frac{1}{x}\frac{\partial}{\partial x}\left(xy\frac{\partial y}{\partial x}\right). \tag{6}$$

The following boundary conditions apply for Eq. (6):

$$\left.\frac{\partial y}{\partial x}\right|_{x=0}=0;\ y|_{x=1}=0. \tag{7}$$

The solution of Eq. (6) is written as a series in even powers of x:

$$y=y_0+y_1x^2+\ldots+y_kx^{2k}+\ldots \tag{8}$$

(it is evident that the coefficients of the odd powers must vanish because $\left.\frac{\partial y}{\partial x}\right|_{x=0}=0$).

We obtain the following system of equations for the functions $y(\tau)$:

$$\left.\begin{aligned}&\frac{\partial y_0}{\partial\tau}=4y_0y_1;\\&\frac{\partial y_1}{\partial\tau}=8\,(2y_0y_2+y_1^2);\\&\ldots\ldots\ldots\ldots\\&\frac{\partial y_k}{\partial\tau}=2\,(k+1)^2\sum_{i=0}^{k+1}y_iy_{k+1-i}.\end{aligned}\right\} \tag{9}$$

Now, if $y_0(\tau)$ is given arbitrarily we can obtain all the $y_k(\tau)$. We investigate possible asymptotic forms of the function $y_0(\tau)$.

If $y_0(\tau)\sim e^{-\gamma\tau}$, then $y_k(\tau)$, starting with $k=2$, becomes negative and increases exponentially at large τ. Consequently, for certain values of x, $y_0(x,\tau)$ increases as $\tau\to\infty$; this is not physically reasonable.

Let $y_0(\tau) = a\tau^q$. Then it can be shown by mathematical induction that $y_k(\tau) = \beta_k \tau^{-k(1+q)+q} a^{1-k}$, where β_k is independent of $\underline{a}$. In this case the series in Eq. (8) can be reduced to the form of a function:

$$y(x, \tau) = a\tau^q G\left(\frac{x^2}{a\tau^{q+1}}\right), \tag{10}$$

where G is some unknown function.

If $q \neq -1$, solutions of the type given in (10) can not satisfy the boundary conditions. We find

$$y|_{x=1} = a\tau^q G\left(\frac{1}{a\tau^{q+1}}\right) = 0 \text{ for all } \tau, \text{ i.e., } y \equiv 0.$$

If $q = -1$

$$y(x, \tau) = \frac{a}{\tau} F\left(\frac{x^2}{a}\right). \tag{11}$$

The function F can be found from Eqs.(9) in the form of a series:

$$F = 1 - \frac{\lambda^2}{4} - \frac{\lambda^4}{64} - \frac{\lambda^6}{288} - \frac{\lambda^8}{1024} - \ldots, \tag{12}$$

where

$$\lambda^2 = \frac{x^2}{a}.$$

This function is positive at small values of λ and vanishes when $\lambda = \lambda_0$.

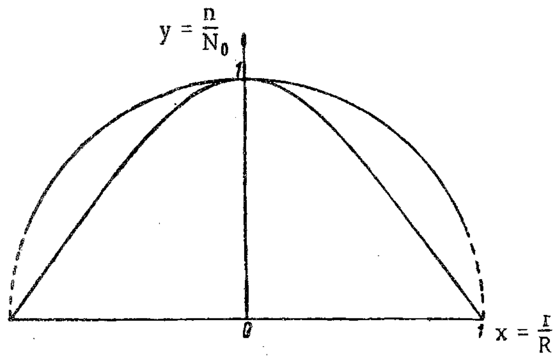

Fig. 2. Aysmptotic form of the radial density distribution in a plasma diffusion across a magnetic field: for the upper curve $D_\perp = \alpha n$, for the lower curve $D_\perp$ = const.

When F = 0 the series in (12) converges very slowly so that λ_0 can not be found directly from it. In order to find λ_0 we substitute $y = (1/\tau) F(x^2)$ in Eq. (6). The equation for F is then

$$-F = \frac{1}{2x}\frac{d}{dx}\left(x\frac{d}{dx}F^2\right). \tag{13}$$

We multiply this equation by 2x, integrate between the limits of 0 and $\underline{x}$, divide by $\underline{x}$, and integrate over the limits $(0, \lambda_0)$. We find

$$-2\int_0^{\lambda_0}\frac{dx}{x}\int_0^x xF\,dx = F^2|_0^{\lambda_0} = -1.$$

Substituting F in the form of the series in (12) we obtain an equation for λ_0:

$$-1 + \frac{\lambda_0^2}{2} - \frac{\lambda_0^4}{32} - \frac{\lambda_0^6}{1152} - \frac{\lambda_0^8}{9216} - \ldots = 0. \tag{14}$$

This series converges rapidly. Retaining the first three terms we find $\lambda_0^2 = 2.4$. The higher terms give a correction of approximately 1%.

To satisfy the boundary conditions we write $a = 1/\lambda_0^2$. Finally,

$$y(x, \tau) = \frac{1}{\lambda_0^2\tau} F(\lambda_0^2 x^2). \tag{15}$$

Investigation of the function F (x) shows that near the points x = 1 it is of the form $F \simeq \mathrm{const}\sqrt{1-x}$ so that $\left.\frac{\partial F}{\partial x}\right|_{x=1} = \infty$. This relation has a simple physical meaning.

Near x = 1 the density vanishes and for a nonvanishing flux the density gradient must become infinite.

Evidently, a wide class of physically reasonable solutions of the boundary value problem (7) for Eq. (6) will have the same asymptotic behavior. This follows from the fact that in regions of space where n is large there is a rapid equalization of the density whereas at the walls there is an infinite gradient.

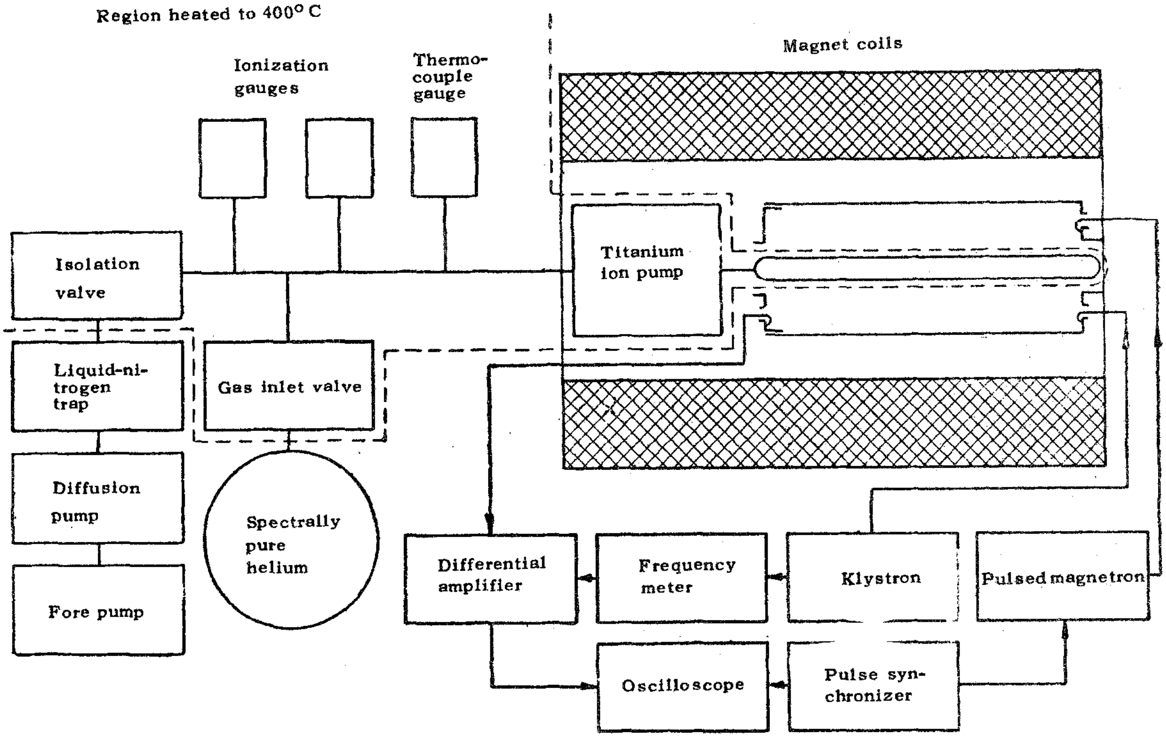

Fig. 3. Block diagram of the apparatus.

We note that the function

$$y(x, \tau) = \frac{1}{\lambda_0^2 \tau + C} F(\lambda_0^2 x^2),$$

where C is an arbitrary constant, is an exact solution of Eq. (6) with the boundary conditions given in (7) and the initial condition

$$y(x, \tau)|_{\tau=0} = \frac{1}{C} F(\lambda_0^2 x^2).$$

In particular, if C = 1 we obtain a function that describes the diffusion process when the initial density at the axis of the tube is N_0. In this case the change in density at the axis of the tube $n_0(t)$ is given by

$$\frac{1}{n_0(t)} - \frac{1}{N_0} = \frac{at}{\Lambda^2}, \tag{16}$$

where $\Lambda^2 \approx R^2/2.4$ is the square of the diffusion length.

In Fig. 2 we show the asymptotic form of the radial density distribution computed from Eq. (12) for the case of diffusion across a magnetic field caused by Coulomb collisions. For purposes of comparison, in the same figure we show the density distribution when the diffusion coefficient is independent of density [the lower curve $y = J_0(\varkappa_{01} x)$].

Experimental Method

In contrast with measurements of the diffusion coefficient (caused by collisions of charged particles and neutrals), for the determination of which it is sufficient to know the relative plasma density, in the present case the accuracy in the measurement of the diffusion coefficient depends on the accuracy of the determination of the absolute density. To determine the mean density in the present experiment the discharge tube is placed in a long cylindrical resonator in which the TH_{019} mode is excited. The resonator simultaneously serves for producing the plasma by means of a microwave pulse.

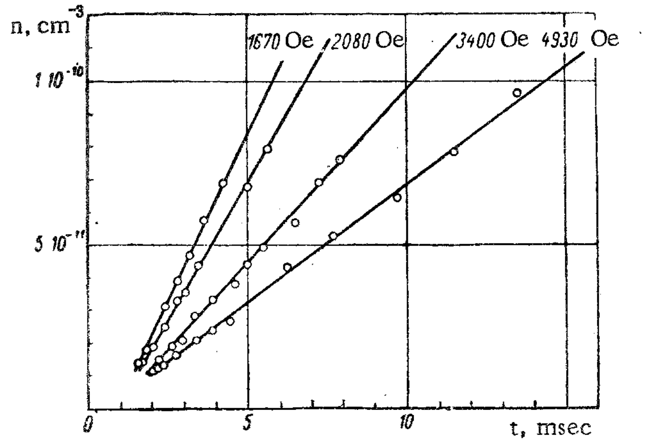

Fig. 4. Typical experimental curves.

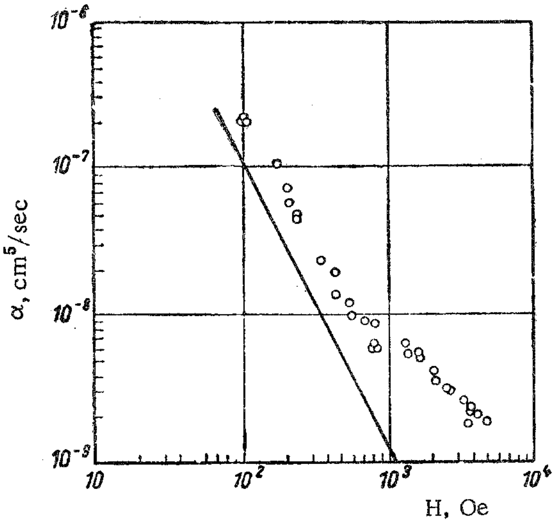

Fig. 5. Variation of diffusion coefficient with magnetic field; the circles denote the experimental points while the solid line is predicted by the Spitzer-Braginskii theory [5, 6].

The long resonator length allows us to neglect end effects in analyzing the interaction of the microwave with the plasma column and also yields the possibility of obtaining a uniform plasma over the length of the tube. A diagram of the apparatus is shown in Fig. 3 and a detailed description has been given in [7].

The determination of the transfer coefficient from the shift of the resonant frequency of the cavity as a function of plasma density requires analytic investigation of the interaction of the TH_{01l} mode with a plasma in a magnetic field.

However, because the plasma radius in the present experiments is small compared with the cavity radius, the transverse components of the electric field can be neglected. This procedure is valid when $k_{\parallel} < k_{\perp}$ and $\omega \neq \omega_{ce}$, where $k_{\parallel} = l\pi/L_0$; $k_{\perp} = \varkappa_{01}/R_0$ (L_0 and R_0 are the length and radius of the cavity; $\varkappa_{01}$ is the first root of the zero-order Bessel function); ω is the resonance frequency of the cavity; ω_{ce} is the electron cyclotron frequency.

In this case the shift of the resonant frequency of the cavity in the presence of plasma is given by the usual perturbation-theory expression [8]:

$$\Delta\omega = -\frac{1}{2\omega}\frac{\int\limits_{\text{plasma}} \omega_0^2 E^2\,dV}{\int\limits_{\text{cavity}} E^2\,dV}, \qquad (17)$$

where ω_0 is the plasma frequency.

Substituting in Eq. (17) the expressions for the density distribution (12) and the electric field distribution in the TH_{01l} mode, we have

$$\Delta\omega = -\frac{4e^2 n_0}{m\omega}\frac{k_\perp^2}{k_\perp^2+k_\parallel^2}\frac{\int\limits_0^R\int\limits_0^{2\pi}\int\limits_0^{L_0} J_0^2(k_\perp r)\cos^2(k_\parallel z)\left[1-\frac{3}{5}\frac{r^2}{R^2}-\frac{3}{80}\frac{r^4}{R^4}-\dots\right] r\,dr\,d\varphi\,dz}{L_0 R_0^2 J_1(x_{01})}. \tag{18}$$

Integrating with the experimental parameters $R_0 = 4.25$ cm, $L_0 = 75$ cm, $R = 0.8$ cm and $\omega = 2\pi \cdot 3.24 \cdot 10^9$ sec^{-1} we find

$$n_0 = 2.1 \cdot 10^2 \Delta\omega. \tag{19}$$

Discussion of Experimental Results

The measurements of density in the afterglow were carried out in the range 10^{10}-10^{11} cm^{-3}. The lower limit of the measured densities is determined by the lower limit for Coulomb diffusion (cf. Fig. 1) while the upper limit is determined by the time required for the electron gas to cool and the capabilities of the experimental apparatus. The experiments were carried out in a glass tube filled with spectrally pure helium at pressures $5 \cdot 10^{-2} - 2 \cdot 10^{-1}$ mm Hg. The system was first outgassed at a temperature of 400°C for a long period of time after which the residual vacuum was of order 10^{-9} mm Hg. In addition, the walls were processed by a microwave discharge operating at the electron cyclotron resonance.

Several experimental curves from which the diffusion coefficients were determined are shown in Fig. 4. Inasmuch as the experimentally observed time dependence of the density is in good agreement with Eq. (16) it is evident that the process responsible for the removal of charged particles has a quadratic dependence on density.

The deviation from Eq. (16) at the beginning of decay (extrapolation of the curves kt = 0 gives $N_0 < 0$) is due to the fact that in the initial stage of decay in the plasma caused by microwave breakdown there are other mechanisms in operation which have an important effect on the density distribution; these include cooling of the electron gas and other factors which are not taken into account in Eq. (16).

In Fig. 5 the points show the measured values of the coefficients α as a function of the magnetic field. The solid line in the same figure corresponds to Eq. (3) for the coefficient of diffusion in a fully ionized gas. It follows from an analysis of the experimental results that up to the field values of 700-800 Oe we have a $D_{ei}^{\perp} \sim 1/H^2$ dependence. The deviation in the absolute value of the diffusion coefficient from the theoretical value is possibly due to systematic errors in the determination of density. An estimate of the errors in the experimental method gives a value of ± 30%, which is somewhat smaller than the observed discrepancy.

In the region from 1000 to 5000 Oe the dependence approaches 1/H. The coefficient for volume recombination of electrons and ions as well as the coefficient for Coulomb diffusion are directly proportional to the density of charged particles. Hence, the experimentally observed coefficient for the removal of particles is actually the sum $\alpha = \alpha_{rec} + \alpha_{dif}/\Lambda^2$. Starting from these considerations, the deviation in the coefficient α at $H > 1000$ Oe from the theoretical dependence ($\alpha_{dif} \sim 1/H^2$) can be easily attributed to volume recombination (for example of the form $He_2^+ + e$) if it were not so close to the 1/H dependence on magnetic field. Under these conditions one must make some artifical assumptions as to the dependence of α_{rec} on magnetic field.

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