# TRANSIENT BEHAVIOR OF A SYSTEM OF PARAMETRICALLY EXCITED SPIN WAVES

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A theoretical computer-assisted investigation was made of the transient effects that arise when waves are excited parametrically. It was established that, in the framework of a previously formulated "S model" [1], the wave system relaxes when "pumping" is switched on from the thermal level to a stationary state $n_k \sim \delta(k - k_0)$ on some surface in k space defined by the condition of external stability. During the last stage of the relaxation, the distribution $n_k(t)$ has the form of a Gaussian packet with width proportional to $\sim 1/\sqrt{t}$. The relaxation process was studied in detail by means of a computer; it depends strongly on the amount by which the parametric excitation exceeds the threshold and the relations between the coefficients of the Hamiltonian. When there is no internal stability with respect to the zeroth mode, the total amplitude of the waves grows unrestrictedly and the packet as a whole is "ejected" from the resonance surface.

In the present investigation we studied the transient effects that arise in ferrodielectrics when a pumping magnetic field $h(\mathbf{r}, t) = h\exp(-i\omega_p t)$, which is periodic in time and homogeneous in space, is applied to them. It is well known that if the pumping amplitude is greater than the threshold $h_c$ defined by the damping of the waves, pairwise coupled spin waves are parametrically excited in the ferrodielectric. They increase exponentially in time until nonlinear effects become important. The consistent description of the further evolution of the system of parametric spin waves is a many-body problem.

The nonlinear theory of the stationary state of parametrically excited spin waves has been developed by Zakharov, L'vov, and Starobinets [1]. The theory is formulated in the classical Hamiltonian formalism and is based on the replacement of the interaction Hamiltonian of the system of parametric spin waves by the part that is diagonal in pairs:

$$\mathscr{H}_{\text{int}} = \sum_{\mathbf{k},\,\mathbf{k}'} \left\{ T_{\mathbf{k}\mathbf{k}'} a_{\mathbf{k}} a_{\mathbf{k}}^* a_{\mathbf{k}'} a_{\mathbf{k}'}^* + \frac{1}{2} S_{\mathbf{k}\mathbf{k}'} a_{\mathbf{k}} a_{\mathbf{k}}^* a_{\mathbf{k}'} a_{-\mathbf{k}'}^* \right\}. \quad (1)$$

Here the canonical variables $a_{\mathbf{k}}$ are the complex amplitudes of the spin waves; the coefficients $T_{\mathbf{k}\mathbf{k}'}$ and $S_{\mathbf{k}\mathbf{k}'}$ describe the nonlinear shift of the frequency and the parametric interaction of the spin waves with one another. It can be shown that the points in the $\mathbf{k}$ space at which the amplitude of the pairs $n_{\mathbf{k}} = \langle a_{\mathbf{k}} a_{\mathbf{k}}^* \rangle$ in the stationary state is nonvanishing are distributed on surfaces in the $\mathbf{k}$ space. The condition of external stability (against the formation of new pairs) [1] uniquely determines the equation of the surface for the possible stationary states:

$$\tilde{\omega}_{\mathbf{k}} \equiv \omega_{\mathbf{k}} - \frac{\omega_p}{2} + 2\sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'} \ldots \sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'} \quad (2)$$

where $\omega_{\mathbf{k}}$ is the dispersion law of the spin waves.

For complete stability the condition of internal stability (against a change of the intrinsic amplitudes and phases) must also be satisfied.

In the present investigation the evolution in time of parametric spin waves was studied for the simple model of an isotropic interaction with the pumping $V_k$ = const [see the Hamiltonian (5)]. This situation is realized when there is parametric excitation of spin waves in antiferromagnets [2]. With a good accuracy of the order of the ratio of the energy of the dipole–dipole interaction to the energy of crystallographic anisotropy one can assume $V_k$ = const for parallel pumping in uniaxial ferromagnets with anisotropy of the easy-plane type [3]. The deductions of the present paper are also qualitatively correct for parallel pumping in cubic ferromagnets (for example, YIG), in which $V_k = V_\theta$ has axial symmetry; this is because up to excess $h/h_c$ of order 6-8 dB the pairs are grouped near $\theta = \pi/2$ [4].

The nonlinear equations (9) describing the time evolution of the parametric spin waves for our model are formulated in Sec. 1; in Secs. 2 and 3 it is shown that in the stable case $S(2T + S) > 0$ the parametric spin waves relax when the pumping is switched on from the thermal level $n_k = n_0$ to the stationary state:

$$\left.\begin{aligned} n_{\mathbf{k}} &= n_{|\mathbf{k}|} \sim \delta(\mathbf{k}-\mathbf{k}_0),\\ N &= N_0 = \sum_{\mathbf{k}} n_{\mathbf{k}} = \frac{\sqrt{(hV)^2-\gamma^2}}{|S|}\,. \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \psi_{\mathbf{k}} &= \psi_0, \quad \sin\psi_0 = \frac{\gamma}{hV};\\ \tilde{\omega}_{\mathbf{k}_0} &= 0, \quad \text{i.e.,}\ \omega_{\mathbf{k}_0} - \frac{\omega_p}{2} + 2TN_0 = 0. \end{aligned}\right\} \tag{4}$$

The relaxation process takes place in three stages – linear, nonlinear, and asymptotic. In the third stage the distribution $n_k(t)$ has the form of a Gaussian packet with half-width proportional to $\sim 1/\sqrt{t}$ (14), which is a self-similar solution (11) of Eqs. (9) for large t. At small excesses ($hV - \gamma \ll \gamma$, see Sec. 2) self-similarity of $n_k(t)$ obtains at all times. A numerical experiment showed that for $hV - \gamma \ll \gamma$ the distribution $n_k(t)$ does indeed always have a Gaussian form and it revealed important differences in the behavior of the parametric spin waves in the nonlinear stage at high excesses (Sec. 3).[1] In particular, the distribution $n_k(t)$ may have several maxima and the total amplitude $N(t)$ oscillates as the stationary value $N_0$ is reached with frequency $\Omega_0 = 2[(2T + S)/S]^{1/2}hV$ and decay constant $\gamma$.

These oscillations correspond to shock excitation of collective degrees of freedom in the system of interacting parametric spin waves.

In Sec. 4 it is shown that if there is no internal stability against the zeroth mode the amplitude $N(t)$ increases unrestrictedly and the packet as a whole is "ejected" from the resonance surface.

It should be noted that the evolution in time of parametric spin waves has been studied in [5]; however, in this paper no allowance is made for the interaction of the spin waves with one another and it is assumed that the amplitudes of the parametric spin waves are restricted because of their feedback influence on the pumping.

## 1. BASIC EQUATIONS

The equations of motion for the amplitudes $n_k$ and phases $\psi_k$ of the pairs,

$$n_{\mathbf{k}} = \langle a_{\mathbf{k}} a_{\mathbf{k}}^* \rangle, \quad \langle a_{\mathbf{k}} a_{-\mathbf{k}} e^{i\omega_p t} \rangle = n_{\mathbf{k}} e^{i\psi_{\mathbf{k}}},$$

in the framework of the Hamiltonian

$$\mathscr{H} = \sum \{\omega_{\mathbf{k}} a_{\mathbf{k}} a_{\mathbf{k}}^* + (hV_{\mathbf{k}} a_{\mathbf{k}} a_{\mathbf{k}} + \text{c.c.})\} + \tilde{\mathscr{H}}_{\text{int}} \tag{5}$$

have the form

$$\left.\begin{aligned} \frac{1}{2}\frac{\partial n_{\mathbf{k}}}{\partial t} &= n_{\mathbf{k}}[-\gamma + \operatorname{Im}\mathscr{P}_{\mathbf{k}}],\\ \frac{1}{2}\frac{\partial \psi_{\mathbf{k}}}{\partial t} &= \tilde{\omega}_{\mathbf{k}} + \operatorname{Re}\mathscr{P}_{\mathbf{k}}, \end{aligned}\right\} \tag{6}$$

where

$$\tilde{\omega}_{\mathbf{k}} = \omega_{\mathbf{k}} - \frac{\omega_p}{2} + 2\sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'} \tag{7}$$

is the frequency difference renormalized by the interaction of the waves, and

$$\mathscr{P}_{\mathbf{k}} = \Big(hV_{\mathbf{k}} + \sum_{\mathbf{k}'} S_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'} e^{-i\psi_{\mathbf{k}'}}\Big) e^{i\psi_{\mathbf{k}}} \tag{8}$$

is the self-consistent total pumping. These equations are derived in [1], which also contains an investigation of their stationary solutions.

We shall study the behavior of the system of parametrically excited waves in time for the isotropic model $V_k$ = const, when $S_{kk'}$ and $T_{kk'}$ de-

[1]The calculations were carried out on the BÉSM-6 computer of the Computational Center of the Siberian Branch of the Academy of Sciences of the USSR.

pend only on the angle between **k** and **k'**. At the same time it is natural to consider only isotropic distributions $n_{\mathbf{k}}$ in the **k** space and for these, Eqs. (6) simplify:

$$\left.\begin{aligned} \frac{1}{2}\frac{\partial n_{\mathbf{k}}}{\partial t} &= n_{\mathbf{k}}\left\{-\gamma + hV\sin\psi_{\mathbf{k}} + S\sum_{\mathbf{k}'} n_{\mathbf{k}'}\sin(\psi_{\mathbf{k}}-\psi_{\mathbf{k}'})\right\}, \\ \frac{1}{2}\frac{\partial\psi_{\mathbf{k}}}{\partial t} &= \omega_{\mathbf{k}} - \frac{\omega_p}{2} + 2T\sum_{\mathbf{k}'} n_{\mathbf{k}'} \\ &+ S\sum n_{\mathbf{k}'}\cos(\psi_{\mathbf{k}}-\psi_{\mathbf{k}'}) + hV\cos\psi_{\mathbf{k}}. \end{aligned}\right\} \qquad (9)$$

The stable stationary solution of these equations has the form (3).

## 2. SMALL EXCESS ABOVE THE INSTABILITY THRESHOLD

In this section we shall study the establishment of the stationary state (3) from the thermal noise level $n_{\mathbf{k}} = n_0$ for small excesses $hV - \gamma \ll \gamma$. It is obvious that as long as $N = \sum \bar{n}_{\mathbf{k}} \ll N_0$, the amplitudes of the pairs will grow in accordance with the linear theory with the growth rate $hV - \gamma$. At the same time, a narrow packet with width $\Delta\omega \simeq (hV - \gamma) \ll \gamma$ is excited in the **k** space. Its subsequent fate is described by Eqs. (9). It can be seen from these equations that the relaxation times of the amplitudes and the phases are of order $1/(hV - \gamma)$ and $1/\gamma$, respectively; we may therefore assume that the phases $\psi_{\mathbf{k}}$ follow the amplitudes adiabatically, i.e., we can neglect $\partial\psi_{\mathbf{k}}/\partial t$.

Because the packet is narrow, $\Delta\omega \ll \gamma$, we can expand the trigonometric functions in Eqs. (9) in series and represent Eqs. (9) in the form

$$-\frac{1}{f}\frac{\partial f}{\partial\tau} = x^2 + x(2\alpha-1)\left(\int f\,dx' - 1\right) + \alpha\int f\,dx' - \alpha + \left[\int f\,dx' - 1\right]^2\alpha^2 + \int x' f\,dx', \qquad (10)$$

where $\alpha = (2T + S)/S$; the dimensionless variables are

$$f(x,\tau) = \frac{n_{\mathbf{k}}(\tau)}{N_0}, \quad x = \frac{\omega_{\mathbf{k}} - \omega_{\mathbf{k}_0}}{2SN_0}, \quad \tau = \left(\frac{2SN_0}{\gamma}\right)^2 \gamma t.$$

Note that in Eq. (10) the dependences on the excess $hV - \gamma$ disappear and the parameters of the system occur only in the ratio $T/S$. Equation (10) has the self-similar solution

$$f(x,\tau) = A(\tau)\sqrt{\frac{\tau}{\pi}}\exp-\left(\frac{x - x_0(\tau)}{d(\tau)}\right)^2, \qquad (11)$$

where $A(\tau) = \int f dx$, the total intensity, $x_0(\tau) = A^{-1}\int x f dx$, the position of the center of gravity of the packet, and $d(\tau)$, its width, satisfy the equations

$$\left.\begin{aligned} &\frac{1}{A}\frac{dA}{d\tau} + \frac{1}{2\tau} - \frac{d(\tau x_0^2)}{d\tau} \\ &= -Ax_0^2 - \alpha A(A-1)[\alpha(A-1)+1], \\ &\frac{d(\tau x_0)}{d\tau} = -(A-1)\left(\alpha - \frac{1}{2}\right), \\ &d(\tau) = \frac{1}{\sqrt{\tau}}. \end{aligned}\right\} \qquad (12)$$

These equations have the following asymptotic behavior for $\tau \gg 1$:

$$\left.\begin{aligned} x_0 = -\frac{1}{2\tau}, \quad A - 1 = -\frac{1}{2(4\alpha-1)\tau^2}, \\ d(\tau) = \frac{1}{\sqrt{\tau}}. \end{aligned}\right\} \qquad (13)$$

Thus, for small excess above the threshold an arbitrary pair distribution function $n_{\mathbf{k}}$ relaxes to the stationary state (9) with $\delta$-function form in accordance with the power law

$$\left.\begin{aligned} N_0 - N &\sim N_0\frac{1}{(hV-\gamma)^2 t^2}, \\ \omega_{\mathbf{k}} - \omega_{\mathbf{k}_0} &\sim \frac{\gamma}{\sqrt{\gamma(hV-\gamma)t}}, \\ \omega_{\mathbf{k}}^2 - (\bar{\omega}_{\mathbf{k}})^2 &\sim \gamma^2\sqrt{\frac{hV-\gamma}{\gamma}}\frac{1}{\sqrt{\gamma t}}. \end{aligned}\right\} \qquad (14)$$

For a detailed study of the nonstationary behavior of the system for arbitrary $hV$, $S$, and $T$ we solved Eqs. (9) numerically. The accuracy of the calculations was verified as follows.

As is readily seen, Eqs. (9) for $\gamma = 0$ have an energy integral. The accuracy of the calculation was tested by testing the energy conservation; the loss of the integral during a time much greater

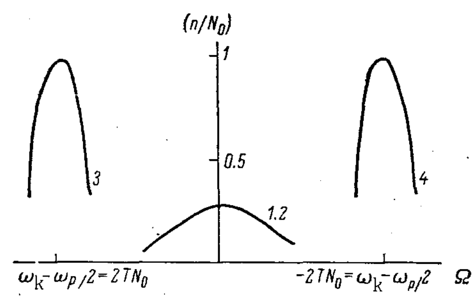


Fig. 1. Distribution function $n_k(t)$ for $hV = 1.4\gamma$. 1, 2) Linear stage ($\alpha = 1/6$, $\alpha = 1$); 3, 4) asymptotic stage ($\alpha = 1/6$, 1, respectively).

than all the characteristic times of the problem, $\sim 150(1/\gamma)$, was $\sim 1.5\%$ for the chosen parameters of the calculation.

In Fig. 1 we show the distribution function $n_k(t)$ for two successive instants of time and the values $\alpha = 1/6$ and 1 of the parameter. The initial distribution was taken to be homogeneous, $hV = 1.4\gamma$. Initially, the waves grow exponentially and the maximum of $n_k(t)$ is situated on the surface $\omega_k = \omega_p/2$, as follows from the linear theory. Of course, the form of the function $\Sigma n_k$ does not depend on the parameter $\alpha$. Subsequently, when $\Sigma n_k$ is not small compared with $N_0$, the behavior of $n_k(t)$ depends strongly on the ratio of the coefficients S and T describing the interaction of the waves; namely, for $T = 0$ the packet $n_k$ continues to grow and contract, remaining on the surface $\omega_k = \omega_p/2$. If $T \neq 0$, the packet grows and moves as a whole, being deformed somewhat, to the surface $\omega_k + 2TN_0 = \omega_p/2$ (i.e., to larger or smaller values of k, depending on the sign of T). When the maximum of the packet $n_k(t)$ is near this surface, the asymptotic contraction of the packet begins. Note that the form of the curve $\ln n_k(t)$ (Fig. 1) for large t is nearly that of a parabola, which confirms the transition to the self-similar solution (11) – a Gaussian packet.

Figure 2 shows the dependence of the integrated amplitude $N(t) = \Sigma n_k(t)$ of the pairs on the time. It can be seen that the process by which the stationary state is reached from the thermal level when the parametric pumping is switched on occurs in three stages: In the first – linear – stage the amplitude grows exponentially and there is no dependence on the nonlinear characteristics T and S of the system of waves. It is therefore natural that the two curves for N(t) (for $\alpha = 1/6$, 1) coincide in this stage.

In the second – nonlinear – stage there is an important mutual nonlinear shift of the frequency and parametric interaction of pairs of waves. At the same time, there is not yet the compensation, characteristic for the stationary state in the "S model," of the damping of the waves $\gamma_k$ by the total pumping $P_k$. The behavior of the system in this stage is at its most complicated and is amenable only to numerical simulation. It turns out that for $T < 0$ the integrated amplitude passes through a maximum, whereas for $T > 0$ it increases monotonically. In the third – asymptotic – stage $||P_k| - \gamma_k| \ll \gamma_k$ and there is a slow approach to the steady state (3) described by the "S model." Note that the sign of the difference $N - N_0$ in the

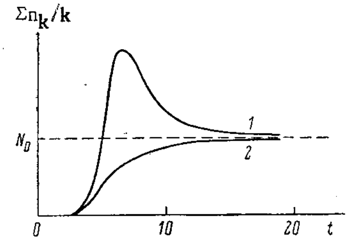


Fig. 2. Total amplitude N as a function of the time for $hV = 1.4\gamma$. $\alpha$: 1) 1/6; 2) 1.

numerical experiment (Fig. 2) is the same as that which follows from the analytic asymptotic behavior (13).

## 3. LARGE EXCESSES ABOVE THE INSTABILITY THRESHOLD

The numerical experiment revealed important qualitative differences in the transient behavior of the system for small ($hV - \gamma \ll \gamma$) and large ($hV - \gamma \gg \gamma$) excesses above the threshold (Figs. 3 and 4 for $hV = 4\gamma$) during the second – nonlinear – stage of the transient process.

The packet $n_k(t)$ does not behave as a single whole; when the amplitude $\Sigma n_k$ is not small compared with $N_0$, a second maximum of the function

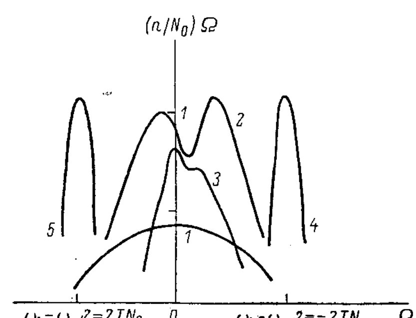


Fig. 3. Distribution function $n_k(t)$ for $hV = 4\gamma$. 1) Linear stage ($\alpha = 1/6$, $\alpha = 1$); 2, 3) nonlinear stage ($\alpha = 1/6$, $\alpha = 1$, respectively); 4, 5) asymptotic stage ($\alpha = 1/6$, 1, respectively).

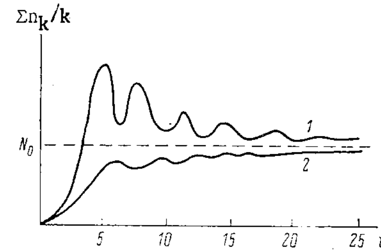


Fig. 4. Total amplitude N as a function of the time for $hV = 1.4\gamma$. $\alpha$: 1) 1/6; 2) 1.

$n_k(t)$ "goes up" at the point where $\tilde{\omega}_k = 0$ at a given instant of time. The amplitude of the second maximum then grows and that of the first decreases. In Fig. 3 for $t = 6(1/\gamma)$ the second maximum is already greater than the first. In addition, as can be seen from Fig. 4, the total amplitude oscillates, approaching $N_0$. In the limiting case of a nondissipative medium, when $\gamma/LV \to 0$, the system of parametric spin waves does not reach a stationary state at all, but oscillates forever. This is because for $\gamma = 0$ an integral of the motion exists:

$$\mathcal{H} = 2\left\{\sum_{\mathbf{k}}\left(\omega_{\mathbf{k}} - \frac{\omega_p}{2}\right)n_{\mathbf{k}} + \sum_{\mathbf{kk'}} T_{\mathbf{kk'}}n_{\mathbf{k}}n_{\mathbf{k'}} + \sum_{\mathbf{k}} hV_{\mathbf{k}}n_{\mathbf{k}}\cos\psi_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{kk'}} S_{\mathbf{kk'}}n_{\mathbf{k'}}\cos(\psi_{\mathbf{k}} - \psi_{\mathbf{k'}})\right\},$$

which can be interpreted as the Hamiltonian for the canonical variables $n_k$ and $\psi_k$. It is readily seen that Eqs. (9) are obtained by varying $\mathcal{H}$ in accordance with the rule

$$\frac{\partial n_{\mathbf{k}}}{\partial t} = -\frac{\delta\mathcal{H}}{\delta\psi_{\mathbf{k}}}, \quad \frac{\partial\psi_{\mathbf{k}}}{\partial t} = \frac{\delta\mathcal{H}}{\delta n_{\mathbf{k}}},$$

i.e., $\mathcal{H}$ is indeed an integral of the motion. At the initial instant of time, $\mathcal{H} = 0$ and it does not coincide with the value $\mathcal{H} = -(2T + S)N_0^2$ in the stationary state of the S model, which is therefore unattainable if $\gamma = 0$.

In the last – asymptotic – stage, it can be seen from Fig. 3 that the packet $n_k$ is Gaussian. This one would expect, since this stage is described by Eqs. (10), which have the self-similar solution (11) – a Gaussian packet. Indeed, the condition for (10) to be valid is $S(N - N_0)/\gamma \ll 1$. In accordance with the asymptotic behavior (13), $N(t)$ approaches $N_0$ from above or below, as can be seen in Fig. 4, depending on the value of $T/S$.

Let us now discuss the nature of the oscillations $N(t)$ in the transient regime. To determine the frequency and the decay constant of the oscillations let us consider the behavior of the parametric spin waves for small deviations of the amplitudes and the phases from the stationary values (3). Expanding $S_{kk'}$, $T_{kk'}$, $\delta n_k$, and $\delta\psi_k$ in series in spherical Fourier harmonics, we obtain from (7) an expression for the growth rate $\Gamma_p$ [$\delta n_p$, $\delta\psi_p \sim \exp(\Gamma_p t)$] of the harmonic with number p:

$$\Gamma_p = -\gamma \pm \sqrt{\gamma^2 - 4S_p(2T_p + S_p)N_0^2}. \qquad (15)$$

We consider here the stable situation, when $\mathrm{Re}\,\Gamma_p < 0$ for all p. It can be seen from (15) that for large excesses the system of parametrically excited waves can be characterized by the set of frequencies

$$\Omega_p = 2\sqrt{S_p(2T_p + S_p)}\,N_0, \qquad (16)$$

corresponding to different collective degrees of freedom. Note that the resonance excitation of the zeroth harmonic $\delta n_0 = \sum_{\mathbf{k}} \delta n_{\mathbf{k}}$, $\delta\psi_0 = \delta\psi_{\mathbf{k}}$ has been studied experimentally and theoretically in [6]. The oscillations of $N(t)$ in the transient regime (see Fig. 4) obviously correspond to shock excitation of this mode. The decay constant of these oscillations is, as can be seen from (15), equal to $\gamma$ and the number of oscillations is obviously of order $\Omega_0/\gamma \simeq hV/\gamma$.

## 4. EVOLUTION OF A SYSTEM THAT DOES NOT POSSESS INTERNAL STABILITY

In contrast to the foregoing sections, we shall here study the behavior of parametric spin waves that do not have internal stability against the zeroth harmonic.

A numerical experiment showed that in this case the total amplitude of the pairs $N(t)$ grows unrestrictedly.

Figure 5 shows a typical time dependence ($hV = 1.4\gamma$; $T_0 = -S_0$).

At small excesses the system as a whole is "ejected" from the resonance surface $\omega_k = \omega_p/2$; at large excesses new maxima are formed successively ever further from this surface; a short time $\sim 1/SN_0$ after the beginning of the nonlinear stage the system has departed a distance $SN_0$.

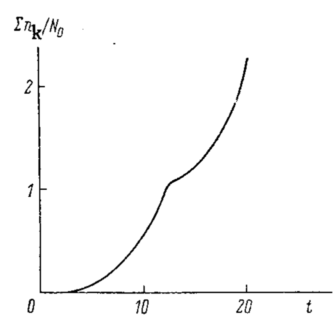


Fig. 5. Total amplitude N as the function of the time for $hV = 1.4\gamma$ ($S_0/T_0 = -1$).

We note finally that the development of instability against the higher harmonics leads to conditions of undamped autooscillations.

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