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OCEANOLOGY

Typical Geometry of Rogue Waves

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INTRODUCTION

Anomalously large surface waves called rogue waves are a rare extreme event on the ocean's surface [1]. The process of formation of anomalously large surface waves is a local concentration of energy in one to two waves [2]. The rogue waves are very interesting for theoretical and practical studies.

In a number of publications (for example, [3, 4]), the authors consider classification of rogue waves. In this work we analyze the results of numerical simulations of sea waves based on the exact equations of hydrodynamics. In these experiments, we repeatedly observed the generation of rogue waves. We prepared an atlas of the geometry of rogue waves on the basis of a large number (more than 3000) of numerical experiments. The majority of them have a characteristic geometry, which allowed us to distinguish the typical profiles of rogue waves and construct their three-parametric regression.

NUMERICAL EXPERIMENTS

We performed numerical modeling of the dynamics of an ideal fluid with a free surface in the domain

$$0 < x < 2\pi, \quad -\infty < y < \eta(x, t).$$

We assumed that 2π periodical conditions were established along the x variable. Zero flow condition was assumed at the bottom ($y = -\infty$). The fluid flow was assumed as potential. Function $\eta(x, t)$ in the model describes the geometry of the free surface at time moment t. In the numerical experiments, we used dynamic equations obtained by Dyachenko written in conformal variables [5]. These equations are equivalent to the Euler equation system and allow us to perform the calculations correctly with a high accuracy [6–8]. These and other versions of equations in conformal variables have been used widely to study rogue waves [9–11].

The formulations of the numerical experiments used in this work are described in [12, 13]. We recorded a rogue wave according to the standard amplitude criterion: a rogue wave is observed at time moment t^* if the following inequality is true:

$$v(t^*) = \frac{H_{\max}(t^*)}{\bar{H}_s(t^*)} \geq v^* = 2.1,$$

where $\bar{H}_s(t^*) = \frac{1}{\Delta T}\int_{t-\Delta}^{t} H_s(\tau)d\tau$ is the mean significant wave height and $H_{\max}(t)$ is the maximum wave height at time moment t. The value $v^* = 2.1$ was selected experimentally; it is used in many publications on rogue waves.

GEOMETRY OF ROGUE WAVES

As a result of a series of numerical experiments, we obtained approximately 3000 geometric profiles of rogue waves corresponding to various wave parameters. We note that the profiles of rogue waves obtained in our experiments were compared with different types of such rogue waves recorded in the sea during the field experiments. A qualitative and quantitative correspondence was established between the realistic records of the waves with the waves modeled in the numerical experiments (see [14]).

In our numerical experiments, we observed three classes of rogue waves: "wall of water," "hole in the sea," and "three sisters." The names of these forms of extreme waves were previously given by the seafarers who encountered rogue waves. "A wall of water" is a solitary anomalous wave with a steep crest. "A hole in the sea" is a deep depression that appears between two neighboring wave crests. "Three sisters" is a packet of several (most frequently, three) sequential anomalously large waves. Approximately 95% of these profiles have almost the same geometry and correspond to the first class: "wall of water." The diagram in Fig. 1 shows the frequency distribution corresponding to the most frequently existing class. Let us find a cubical regression for the left and right sides of waves in the following form:

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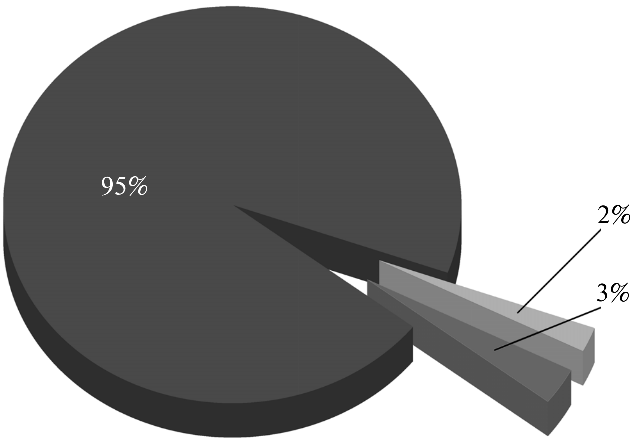

Fig. 1. Relation between the types of rogue waves: “wall of water” 95%; “hole in the sea” 3%; “three sisters” 2%.

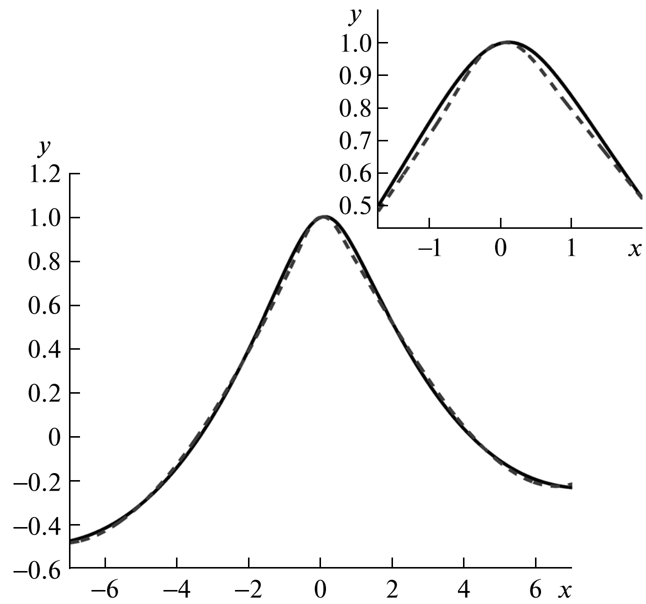

Fig. 2. Approximation of the profile with cubic polynomials.

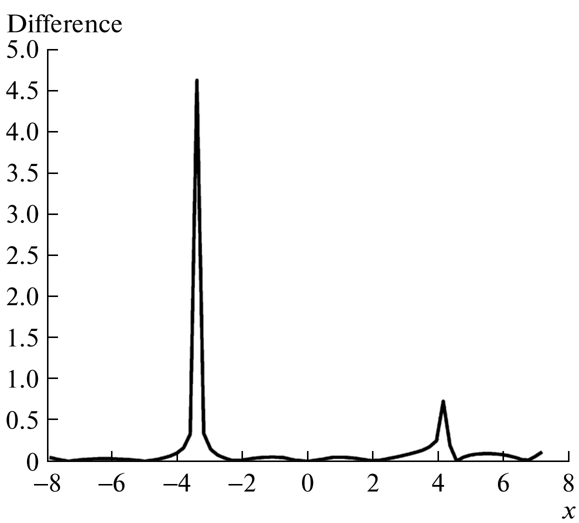

Fig. 3. Relative errors of approximation.

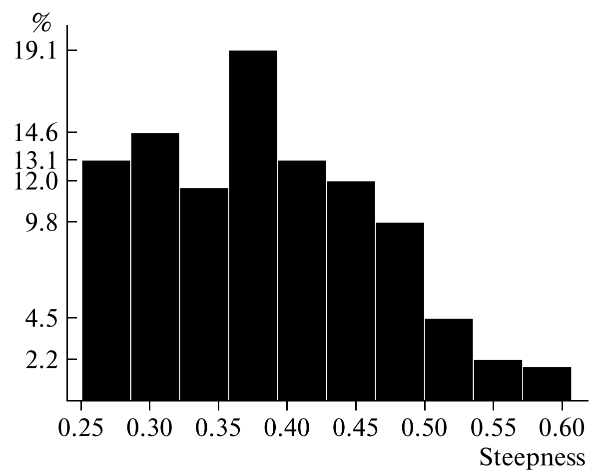

Fig. 4. Distribution of the maximum steepness of rogue waves.

$$P_{\text{left/right}}(x) = a_3x^3 + a_2x^2 + a_1x + a_0.$$

We can consider without loss of generality that the coordinates of the summit are (0, 1); hence $a_0 = 1$. We get the following numerical values for the typical profile:

$$P_{\text{left}} = -0.0012x^3 + 0.0127x^2 + 0.3676x + 1,$$

$$P_{\text{right}} = 0.0028x^3 + 0.0063x^2 + 0.2756x + 1.$$

Figure 2 presents a graph of the profile and the suggested regression, and Fig. 3 shows the relative accuracy of our approximation. The suggested regression with cubic polynomials also appeared effective for the other profiles of rogue waves, which were observed in our experiments.

After the approximation is constructed using analytical functions, one can analyze different geometrical characteristics of rogue waves. In particular, the maximum steepness κ of approximately 95% of the rogue wave profiles is within $\kappa \in [0.25, 0.6]$, while the mean steepness is $\langle\kappa\rangle = 0.38$.

The steepness of the remaining 5% of rogue waves is smaller than 0.25.

Figure 4 gives a diagram of the distribution of the maximum steepness of rogue waves.

We note that the observed extreme waves with a similar form can have a wide range of maximum steepness, from moderate to strongly nonlinear. This indicates that not all waves called rogue waves are really hazardous.

CONCLUSIONS

In this work we presented a quantitative classification of different types of rogue waves based on a large number of results of numerical experiments. It was shown that in the overwhelming majority the geometry of such rogue waves is similar, which makes it possible to realize the regression of the surface form of anomalously high surface waves using cubic polynomials.

The results presented in this paper can be used for construction of the typical profiles of rogue waves and for developing engineering methods to estimate the hazard of such waves for ships and marine constructions.

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