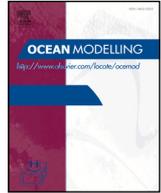




ELSEVIER

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Virtual Special Issue Ocean Surface Waves

Limited fetch revisited: Comparison of wind input terms, in surface wave modeling

Andrei Pushkarev^{b,c,d,*}, Vladimir Zakharov^{a,b,c,d}^a University of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721, USA^b LPI, Leninsky Pr. 53, Moscow 119991, Russia^c Pirogova 2, Novosibirsk State University, Novosibirsk 630090, Russia^d Waves and Solitons LLC, 1719 W. Marlette Ave., Phoenix, AZ 85015, USA

ARTICLE INFO

Article history:

Received 21 February 2015

Revised 14 March 2016

Accepted 20 March 2016

Available online 31 March 2016

Keywords:

Hasselmann equation

Wind–wave interaction

Wave-breaking dissipation

Nonlinear interaction

Self-similar solutions

Kolmogorov–Zakharov spectra

ABSTRACT

Results pertaining to numerical solutions of the Hasselmann kinetic equation (*HE*), for wind driven sea spectra, in the fetch limited geometry, are presented. Five versions of source functions, including the recently introduced *ZRP* model (Zakharov et al., 2012), have been studied, for the exact expression of S_{nl} and high-frequency implicit dissipation, due to wave-breaking. Four of the five experiments were done in the absence of spectral peak dissipation for various S_{in} terms. They demonstrated the dominance of quadruplet wave–wave interaction, in the energy balance, and the formation of self-similar regimes, of unlimited wave energy growth, along the fetch. Between them was the *ZRP* model, which strongly agreed with dozens of field observations performed in the seas and lakes, since 1947. The fifth, the *WAM3* wind input term experiment, used additional spectral peak dissipation and reproduced the results of a previous, similar, numerical simulation described in Komen et al. (1994), but only supported the field experiments for moderate fetches, demonstrating a total energy saturation at half of that of the Pierson–Moscovits limit. The alternative framework for *HE* numerical simulation is proposed, along with a set of tests, allowing one to select physically-justified source terms.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

The motivation, for the research presented in the current paper, was to continue the project of finding a firm scientific foundation for the study of wind driven seas.

The most important step in this direction was made in 1962, by K. Hasselmann (Hasselmann, 1962; 1963; Nordheim, 1928), who proposed the kinetic equation for wind waves description

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss} \quad (1)$$

similar to equations used in condensed media physics since the 1920s (Landau and Lifshitz, 1980), where $\varepsilon = \varepsilon(\omega_k, \theta, \vec{r}, t)$ is the wave energy spectrum, as a function of wave dispersion $\omega_k = \omega(k)$, angle θ , two-dimensional real space coordinate $\vec{r} = (x, y)$ and time t . S_{nl} , S_{in} and S_{diss} are nonlinear, wind input and wave-breaking dissipation terms, respectively. Hereafter, only the deep

water case, $\omega = \sqrt{gk}$ is considered, where g is the gravity acceleration and $k = |\vec{k}|$ is the absolute value of wavenumber $\vec{k} = (k_x, k_y)$.

Eq. (1) is widely accepted in the oceanographic community (Young, 1999; Komen et al., 1994) and has several names. It is called the Boltzmann equation (Komen et al., 1994) (while this is not exactly correct), the energy balance equation (Young, 1999), and the radiation balance equation. We will call it the Hasselmann equation (hereafter *HE*) as a tribute to Hasselmann's pioneering work. At the least, this is consistent with part of the community (Janssen, 2009).

The right side of Eq. (1) consists of three terms. The S_{nl} term is completely known. It was consistently derived from Euler equations and describes quadruplets of waves satisfying resonant conditions

$$\vec{k} + \vec{k}_1 = \vec{k}_2 + \vec{k}_3 \quad (2)$$

$$\omega_k + \omega_{k_1} = \omega_{k_2} + \omega_{k_3}$$

In the papers (Zakharov, 2010; Zakharov and Badulin, 2011) we introduced the following splitting of the S_{nl}

$$S_{nl}(\omega, \theta) = F(\omega, \theta) - \Gamma(\omega, \theta)\varepsilon(\omega, \theta) \quad (3)$$

* Corresponding author.

E-mail address: dr.push@gmail.com (A. Pushkarev).

The explicit expressions for F and Γ are presented in the [Appendix](#). The motivation for this splitting is very simple. The term $F(k)$, for any spectral distribution $\varepsilon(\omega, \theta)$, is an *essentially positive function*. We will soon show that this fact is of fundamental importance.

Kinetic equations similar to the Hasselmann equation are routinely used in different areas of theoretical physics. In all cases, the first and central issue is the description of solutions to the stationary equation

$$S_{nl}(\omega, \theta) = 0 \quad (4)$$

Any solution of this equation can be presented in the form

$$\varepsilon(\omega, \theta) = \frac{F(\omega, \theta)}{\Gamma(\omega, \theta)} \quad (5)$$

As far as $\varepsilon(\omega, \theta) > 0$, for all solutions

$$\Gamma(\omega, \theta) > 0 \quad (6)$$

The function Γ also has another physical sense. In the presence of nonlinear wave ensemble, the dispersion law is undergoing the re-normalization

$$\omega(k) \rightarrow \omega(k) + \Delta\omega(k) \quad (7)$$

The re-normalization has real and imaginary parts. The imaginary part is

$$Im\Delta(\omega) = \frac{1}{2}\Gamma(\omega, \theta) \quad (8)$$

Everybody knows that [Eq. \(4\)](#) has solutions with thermodynamic equilibrium.

There is Maxwell distribution in the kinetic gas theory, and Plank distribution in quantum statistical dynamics. Physicists believed, for a long time, that the thermodynamic equilibrium spectra are unique solutions of [Eq. \(4\)](#). This is certainly true, if the entropy of a solution is finite. However, [Eq. \(4\)](#) has a broad class of solutions with infinite entropy, governed by fluxes of conservative quantities—energy, momentum and wave action.

These solutions are now called KZ (Kolmogorov–Zakharov) solutions and widely used in different areas of physics (see, for instance [Lvov and Newell \(2000\)](#); [C. Connaughton \(2004\)](#); [Galtier \(2000\)](#); [2003](#)). The general theory of KZ solutions is described in the monograph ([Zakharov et al., 1992](#)).

A more advanced development is contained in the paper ([Zakharov, 2010](#)). The discovery of KZ spectra was recognized by the physical community, by awarding a Dirac medal in 2003, for this development.

The first KZ solution was found by Zakharov and Filonenko in 1966 (the English version of [Zakharov and Filonenko \(1967\)](#) was published in 1967). It is the isotropic solution of the stationary Hasselmann [Eq. \(4\)](#) (the details are presented in [Section 4](#)):

$$\varepsilon(\omega) = \frac{\beta_{KZ}}{\omega^4} = C_K \frac{g^{4/3} P^{1/3}}{\omega^4} \quad (9)$$

Here P is the energy flux to the high frequency region. It was soon established, ([Katz and Kontorovich, 1975](#)), that the solution [Eq. \(9\)](#) is only “the tip of the iceberg”. Actually, [Eq. \(4\)](#) has a much bigger class of KZ solutions, outlined in the paper ([Zakharov, 2010](#); [Katz and Kontorovich, 1975](#)). The most interesting and important solutions, governed by fluxes of energy and momentum, are anisotropic. They are not exactly power-like, seeing their ω –dependence deviates from the ω^{-4} law, but only mildly.

Meanwhile, numerous laboratory and field experiments showed that, in the important band of frequency, right behind the spectral peak (approximately for $1.5\omega_p < 3.5\omega_p$), the observed spectra are very close to the ω^{-4} law. Experimental data obtained before 1985 was summarized in the well known paper of Phillips ([Phillips, 1985](#)). Since then, a lot of new data has accumulated (see, for instance, [Romero and Melville \(2010\)](#), [Resio and Perrie \(1989\)](#); [Resio](#)

[et al. \(2004\)](#); [Long and Resio \(2007\)](#), [Gagnaire-Renou et al. \(2011\)](#); [Zakharov et al. \(2014\)](#)). Some other experimental results were cited in the article ([Zakharov, 2015a](#)).

Recall that the exact S_{nl} can be derived, rigorously, from the Euler equation.

Opposite to it, the “source function” S_{in} —the energy income from the wind, and the energy dissipation function S_{diss} , due to wave-breaking, are only known approximately. In the oceanographic community, there is no consensus regarding their form. We discuss these questions in [Sections 2](#) and [3](#) of this paper.

The ambiguity of their proper definitions presents the first major issue for wind wave theory, and hinders development of accurate operational models, as well.

The other important issue is connected with S_{nl} collision term numerical simulation. It is the complex, non-linear, operator, with deep internal symmetries. Several S_{nl} simulation algorithms are available, at the moment, for example: Webb–Resio–Tracy (WRT) ([Tracy and Resio, 1982](#); [Webb, 1978](#)) (also, see important paper ([Resio and Perrie, 1991](#))), Lavrenov ([Lavrenov, 2010](#)) and Masuda ([Komatsu and Masuda, 1996](#)). The Van-Vledder version of the WRT algorithm ([Hwang, 2007](#)) has already been included in the *Wave-watchIII* and *SWAN* models, for more than a decade.

All of the above algorithms provide reliable results, but are too slow to provide simultaneous *HE* solutions of the [Eq. \(1\)](#) for tens of thousands of spatial points, faster than real time, as is required by operational wave forecasting. Because of this, existing operational models use much faster substitutes for S_{nl} , in the form of *DIA* and its analogs. This is not fatal, as long as the number of quadruplet configurations used in *DIA* is large enough. However, what is wrong is the commonly practiced “tuning” of the *DIA* algorithm parameters, in the operational models.

We must stress, however, that we do not discuss the good and bad sides of different modifications of *DIA* models. The only results discussed are those obtained from the numerical algorithm for solving the exact Hasselmann equation. This code is a modification of the WRT algorithm. We hereby call it *XLN*.

We insist that a correct definition of the source function is necessary, and we assert that it is possible to perform these corrections, without new theoretical constructions or new difficult experiments. It is sufficient to use existing experimental data, in a proper way. For 68 years, starting from a well-known work of Sverdrup and Munk ([Sverdrup and Munk, 1947](#)), oceanographers have accumulated a plethora of experimental facts regarding wave growth rate, with respect to winds. Some of those facts were obtained in water tanks, but the most interesting facts come from ocean measurements.

Nowadays, the results of numerous measurements for “fetch limited” field set-ups, where the off shore wind and the waves are quasi-stationary, have been systematized and published ([Badulin et al., 2007](#)).

All of those situations are described by the stationary *HE*

$$\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial x} = S_{nl} + S_{in} + S_{diss} \quad (10)$$

This equation is solved, in the presented research, for different source functions S_{in} and S_{diss} . Five experiments were carried out, for different wind input functions, and their results were compared to known ocean field experimental data. This comparison actively used the fact that the results of those experiments are well described by *Weak Turbulence Theory (WTT)*. This theory is explained, in detail, in the monograph ([Zakharov et al., 1992](#)), and applications of this theory, to ocean experiments, are presented in the publications ([Pushkarev et al., 2003](#); [Zakharov, 2005](#); [Badulin et al., 2005](#); [2007](#); [Gagnaire-Renou et al., 2011](#); [Zakharov et al., 2014](#)).

The possibility of *WTT* application is based on the fact that, in [Eq. \(1\)](#), S_{nl} is the dominant term. This fact can be explained in the

following way. All S_{in} cases considered in the current research are quasi-linear, which means that

$$S_{in} = \gamma(\omega, \theta)\varepsilon(\omega, \theta) \quad (11)$$

$$S_{diss} = -\gamma_{diss}(\omega, \theta)\varepsilon(\omega, \theta) \quad (12)$$

Taking into account S_{nl} splitting, Eqs. (3), (10) take the form

$$\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial \bar{r}} = F(\omega, \theta) - (\Gamma(\omega, \theta) - \gamma(\omega, \theta) + \gamma_{diss}(\omega, \theta))\varepsilon(\omega, \theta) \quad (13)$$

One should note that γ typically has a fairly small value of $10^{-5}\omega_p$, for waves with the frequencies close to the peak frequency ω_p . The value of γ_{diss} does not exceed γ , or waves are not excited at all. Meanwhile, the value of Γ is rather large, as shown by analytic and numeric calculations. It easily exceeds γ , by orders of magnitude (Zakharov, 2010; Zakharov and Badulin, 2011) (see Appendix). Therefore, one can substitute in the first approximation Eq. (13) by conservative equation

$$\frac{\partial \omega_k}{\partial k} \frac{\partial \varepsilon}{\partial x} = S_{nl} \quad (14)$$

which is, indeed, the subject of the WTT study.

It is customary to use “Kitaigorodsky dimensionalization”, where the fetch variable x , total energy E , and peak frequency ω_p are substituted by dimensionless variables

$$\chi = \frac{xg}{u^2}, \quad \varepsilon = \frac{Eg^2}{u^2}, \quad \hat{\omega} = \frac{\omega_p u}{g} \quad (15)$$

All ocean and wave tank measurements show that $\varepsilon(\chi)$ and $\omega(\chi)$ are the power functions of dimensionless fetch χ :

$$\varepsilon = \varepsilon_0 \chi^p \quad (16)$$

$$\hat{\omega} = \omega_0 \chi^{-q} \quad (17)$$

The values of p and q vary in different experiments, but not significantly $0.74 < p < 1$, $0.2 < q < 0.3$. They are connected, with strong accuracy, by the “magic relation”

$$10q - 2p = 1 \quad (18)$$

These facts are explained by WTT (Zakharov, 2005). Conservative kinetic Eq. (14) has a four-parameter family of self-similar solutions (Zakharov, 2005; Pushkarev et al., 2003; Badulin et al., 2005; 2007), for which the “magic relation” is fulfilled exactly.

It was shown in Zakharov et al. (2012); Zakharov (2010) that the non-conservative HE, with the dissipation, localized in short waves, and forcing chosen as the power function

$$\gamma(\omega, \theta) = f(\theta)\omega^s \quad (19)$$

also allows a self-similar solution and preserves the “magic relation” Eq. (18).

All numerical experiments presented in the current paper included short-wave dissipation, but in the “implicit” way: the spectrum at frequencies $f > 1.1$ Hz was forced to Phillips spectrum $\varepsilon_\omega \simeq \omega^{-5}$. The validity of this approach is discussed in Section 3. Similar procedure of matching the spectrum with the powerlike tail at high frequency is routine in the operational models (Tolman, 2013).

Four out of five of those experiments assumed absence of long-wave dissipation. It is assumed hereafter that “long waves” denotes the waves with the characteristic wavenumber close to the spectral peak vicinity. Such experimental set-up contradicts existing tradition, but is justified by obtained results. Four existing wind forcing terms have been checked: ZRP (Zakharov et al., 2012), Chalikov (Tolman and Chalikov, 1996; Chalikov, 1995), Hsiao–Shemdin

(Hsiao and Shemdin, 1983) and Snyder (Snyder et al., 1981a). The only S_{in} term, in the power form, was the ZRP forcing term, and it was only this experiment which showed agreement with the field observation, for which $p = 1$ and $q = 0.3$. The other S_{in} terms lead to the Eqs. (16) and (17), where indices p and q are functions of the dimensionless fetch. It is important to note that the “magic relation” Eq. (18) still holds, as well, which means that corresponding spectra exhibit “local” self-similarity.

The question is: how valid is the claim that the wave energy and the mean frequency behave like powers?

The developed WTT (Weak Turbulent Theory) does not impose limitations on wave energy growth, with the fetch. The maximum length of the dimensionless fetch $\chi \simeq 10^5$ was considered in the experiments of Donelan et al. (Donelan et al., 1985). The corresponding wave energy maximum was $\varepsilon = 4.07 \times 10^{-3}$, without any deviation from the power law.

We should stress that we now speak about a stationary-in-time wave field, and discuss dependence of its characteristics on the fetch, only. In 1964, Person and Moscovitz formulated the hypothesis that on long enough fetches, the wave field becomes statistically homogeneous. According to Young (Young, 1999) this “spatial saturation” occurs at $\chi \simeq 5 \times 10^4$ (see Fig. 5.10 in the cited book) and the energy spectrum is stabilized on the level $\varepsilon \simeq 3.64 \times 10^{-3}$. For that time, it was an important achievement, but we must emphasize that this hypothesis was pure speculative, because Pierson and Moscovitz measured the wave field only at one spatial point. However, recent analysis of numerous experimental data, published in Badulin et al. (2007); Pushkarev et al. (2003); Badulin et al. (2005), does not support the concept of the spatial saturation. Nevertheless, the idea of a “mature sea” was actively supported in the paper (Komen et al., 1984) and became a kind of credo for oceanographers. The WAM3 model was designed specifically to support this idea.

The numerical test of the WAM3 model, using the XNL approach, was performed in the fetch-limited geometry, long ago (see Komen et al. (1994), pp. 229, 254, Fig. 3.7 and Fig. 3.22). It was found that for moderate fetches $10^2 < \chi < 10^3$ this model describes the experimental situation pretty well, however, predicting saturation at the fetch $\chi \simeq 2 \times 10^4$ on the low level $\varepsilon_{Max} \simeq 1.8 \times 10^{-3}$. Our numerical experiments confirmed these results. Moreover, we found that in the “practical fetch” interval $10^2 < \chi < 10^3$, the results of WAM3 coincide with the results obtained via the ZRP model, without any spectral peak dissipation. For larger fetches, the ZRP model demonstrates much better coincidence with field experiments, than the WAM3 model does.

Here one can recall William Okham’s principle “It is futile to do with more things that which can be done with fewer”. Application of this principle leads to excluding the long wave dissipation from consideration, and to the conclusion that the WAM3 model is not consistent enough. It is satisfactory in only one aspect - it passes the ω^{-4} test, explained in Section 4.

The obtained results can be seen as a progressive step toward universal, physically-based, ocean surface wave models, the development of which will require minimal tuning for different ocean conditions. Other perspectives are discussed in the Conclusion.

2. Current state of wind input source terms

Nowadays, the number of existing models of S_{in} is large, but these models lack firm, theoretical, justification. Different theoretical approaches argue with each other. A detailed description of this discussion can be found in the monographs (Young, 1999; Komen et al., 1994), and the papers (Belcher and Hunt, 1998; Chalikov and Makin, 1991; Kudryavtsev et al., 2001; Tolman and Chalikov, 1996; Miles, 1957; Hsiao and Shemdin, 1983; Snyder et al., 1981b; Donelan et al., 1985).

The development of wind wave models had begun as far back as the 1920s, in the well-known works of Jeffreys (Jeffreys, 1924; 1925). His model is semi-empiric and includes an unknown “sheltering coefficient”. All other existing theoretical models are also semi-empiric, with one exclusion – the famous Miles model (Miles, 1957). This model is rigorous, but is related to an idealized situation – the initial stage of wave excitation, by laminar wind, with specific wind profile $U(z)$.

Miles theory application is hampered by two circumstances. First is the fact that the atmospheric boundary layer is the turbulent one, and creating a rigorous, analytic theory of such turbulence is, as of today, an unsolvable problem.

There is the opinion, however, that wind speed turbulent pulsations are small, with respect to horizontal velocity $U(z)$ (Troitskaya et al., 2011; Fabricant, 1976; Nikolaeva and Zymring, 1986; Janssen, 1982) and that they should be neglected in the first approximation (Fabricant, 1976; Janssen, 1982). This does not mean that turbulence is not taken into account, at all. It is suggesting that the role of the turbulence consists in formation of the averaged horizontal velocity profile.

The wide spread opinion is that the horizontal velocity profile is distributed by the logarithmic law

$$U(z) = 2.5u_* \ln \frac{z}{z_0} \quad (20)$$

Here u_* is the friction velocity and z_0 is the roughness parameter

$$z_0 = C_{ch} \frac{u_*^2}{g} \quad (21)$$

where $C_{ch} \simeq 3 \times 10^{-2}$ is the experimental and dimensionless Charnock constant.

One should note that the appearance of an anomalously small constant, not having “formal justification”, is an extremely rare phenomenon in physics. Eqs. (20), (21) mean that the roughness parameter is very small: for typical ocean conditions – wind speed 10 m/s on the height $z = 10$ m we get $z_0 \simeq 5 \times 10^{-4}$ m. Such roughness is only twice the size of the viscous layer, defined from multiple experiments on turbulent wind flow, over smooth metal plates. Notice that the logarithmic law certainly could not work for a height of the order of few centimeters, where capillary effects are essential.

Usage of Eqs. (20), (21) assumes, therefore, that the ocean behaves as smooth metal surface. This is not correct. Horizontal momentum is transferred to the smooth plate, on the surface itself, while in the ocean this process happens differently.

Momentum off-take, from the atmospheric boundary layer, is smoothly distributed over the whole width of the boundary layer and begins from the highest “concurrency layer”, i.e. from the height where the phase speed of the fastest wave matches the horizontal velocity.

Momentum off-take leads to horizontal velocity distribution $U(z)$ depending on time, the wave’s development level, and energy spectrum. Meanwhile, Miles’s instability increment is extremely sensitive to the horizontal velocity profile (there is no wave excitation for the linear profile $U(z)$, in Miles theory, for example). The velocity profile is especially important for slight elevations, on the order of several centimeters, over the water surface, which is almost unknown and difficult for experimental measurements. However, there have been some advances in this direction (Hristov et al., 2003; Troitskaya et al., 2011).

The necessity of taking into account the waves feedback, into the horizontal velocity profile, was understood a long time ago, as seen in the works of Fabricant (Fabricant, 1976) and Nikolaeva et al. (Nikolaeva and Zymring, 1986). This approach was later continued by Janssen (Janssen, 1982) and explained, in detail, in the

monograph Komen et al. (1994) in the form of “quasi-laminar” theory. This theory is lacking.

To consider the theory as self-consistent, even in the approximation of turbulence absence, it is necessary to solve equations describing the horizontal velocity profile $U(z)$, together with the Haselmann equation, describing the energy spectrum evolution. This is not done yet, either.

That fact aside, many theoreticians do not share the opinion about turbulent pulsations insignificance, and consider them as the leading factor. Corresponding *TBH* theory by Townsend, Belcher and Hunt (Belcher and Hunt, 1998) is an alternative to quasi-laminar theory. Both theories are discussed in Donelan et al. (2006).

There is another approach, not connected with experimental analysis – numerical simulation of the boundary atmospheric layer, in the frame of empiric theories of turbulence. It was developed in the works (Chalikov and Makin, 1991; Kudryavtsev et al., 2001; Tolman and Chalikov, 1996; Chalikov, 1995). Since those theories are insufficiently substantiated, the same relates to the correspondingly derived wind input terms.

Across the wide variety of theoretical approaches to defining S_{in} , almost all of them are “quasi-linear” Zakharov and Badulin (2011) where the standard relation (Young, 1999; Komen et al., 1994)

$$\gamma(\omega, \phi) = \frac{\rho_a}{\rho_w} \omega \beta \left(\frac{\omega}{\omega_0}, \phi \right) \quad (22)$$

is being used. Here $\omega_0 = \frac{g}{u}$, where u is the wind speed, defined differently in individual models. The function β is dimensionless and is growing with the growth of $\frac{\omega}{\omega_0}$.

However, even for the models exhibiting the strongest wind input, the value of β belongs to the interval $0 < \beta < 1$, for ξ from the interval $0 < \xi < 3$. In some models (see, for example Tolman and Chalikov (1996)) β becomes negative for the waves propagating faster than the wind, or under large angle, with respect to the wind.

Looking at multiple experimental attempts to define S_{in} , one should notice that they need to be critically analyzed. The criticism is not about the integrity of measurements itself, but about the methodology used, the validity of data interpretation, and the possibility of transferring conclusions made in artificial environments to real ocean conditions.

A significant amount of the experiments, belonging to the so-called “fractional growth method” category, have been performed, through energy spectrum measurement in time, and calculation of the corresponding γ through

$$\gamma(\omega, \phi) = \frac{1}{\varepsilon(\omega, \phi)} \frac{\partial \varepsilon(\omega, \phi)}{\partial t} \quad (23)$$

Eq. (23) is, in fact, the linear part, or just two terms of the *HE* Eq. (1). This method is intrinsically wrong, since it assumes that either advection $\frac{\partial \omega_k}{\partial k} \frac{\partial \varepsilon}{\partial r}$ and nonlinear S_{nl} terms of Eq. (1) are absent altogether, or relation

$$\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial r} = S_{nl} \quad (24)$$

is fulfilled.

The first assumption is simply not correct, since neglected terms are defining ocean conditions.

The second assumption is almost fulfilled, indeed, since the sea is described by the *WTT*. But the terms in Eq. (24) are large with respect to the terms in Eq. (23), and therefore there is no reason to neglect the terms of Eq. (24).

Regarding the “fractional growth method”, we are just citing the single publication by Plant (1982) where, it seems, the author understood its scarcity.

As a matter of fact, it is natural to ask about the spectral correlation function $Q(\omega)$ between the surface elevation η and the wind-induced pressure on the surface P :

$$Q(\omega) = \langle \eta(\omega)P^*(\omega) \rangle \quad (25)$$

where brackets denote ensemble averaging, in Fourier space, and asterisk refers to the complex conjugate.

Unfortunately, the number of such experiments is limited, and not all of them have significant value for describing ocean phenomena. Also, one should not consider the experiments performed in laboratory conditions.

Consider, for example, the set of experiments described in Mastenbroek et al. (1996). These experiments were performed in the wave tank of 40 m length and 1 m depth. The wind was blowing at speeds up to 16 m/s, but they only studied short waves, no longer than 3 m, moving no faster than 3.3 m/s. Therefore, they studied the very short, wave-tail of the function β , in conditions far from those of the ocean. Another problem with flume data is upper physical confinement of the vertical velocity profile. The value of these measurements is not significant.

The same arguments apply to multiple precisely-performed measurements on the Lake George, Australia Donelan et al. (2005). The depth of this lake is, on average, about 1 m. That is why the waves slower than 3.3 m/s can propagate on its surface. The typical wind speed, corresponding to these measurements was 8 – 12 m/s. Therefore, while the results of these measurements are quite interesting, the obtained expression for S_{in} is questionable, because it runs completely against quasi-linear theory. The quasi-linear theory predicts smoothing of the velocity profile $U(z)$, with the wave's development. The wind input growth rate, however, was increasing with the wave's energy level, in the experiments (Young and Babanin, 2006).

After critically analyzing experiments on S_{in} measurements, only three of them deserve attention. Those are the experiments by Snyder et al. (Snyder et al., 1981a), Hsiao et al. (Hsiao and Shemdin, 1983) and Hasselmann et al. (Hasselmann and Bosenberg, 1991). These experiments were performed in the open ocean and measured direct correlations of surface speed change and the pressure. The accuracy was not ideal and the data scatter was significant, presumably due to contemporary technologies. Therefore, their interpretation is quite ambiguous. The fact of this uncertainty was highlighted in the paper (Tolman and Chalikov, 1996). Either way, these experiments produced two well-known formulas for β . Next, we present β 's expressions, for the cases analyzed in the current paper.

For Snyder et al. (Snyder et al., 1981a), Hasselmann-Bosenberg (Hasselmann and Bosenberg, 1991) case

$$\beta_{SHB} = \begin{cases} 0.24(\xi - 1) & \text{for } 1 < \xi < 4 \\ 0, & \text{for } \xi < 1 \end{cases} \quad (26)$$

For Hsiao-Shemdin case (Hsiao and Shemdin, 1983)

$$\beta_{HS} = \begin{cases} 0.12(0.85\xi - 1)^2 & \text{for } \xi \geq 1 \\ 0, & \text{for } \xi < 1 \end{cases} \quad (27)$$

Let us notice that for ZRP case Zakharov et al. (2012)

$$\beta_{ZRP} = 0.05\xi^{4/3} \quad (28)$$

The differences between various S_{in} , corresponding to Eqs. (26)–(28), are significant. For many practical purposes, the spectral peak is located in the interval $1.5 < \xi < 2.5$, where the difference between Snyder and Hsiao-Shemdin functions is huge. Indeed

$$\beta_S(1.5) = 0.12 \quad \beta_{HS}(1.5) = 0.009 \quad (29)$$

$$\beta_S(2.5) = 0.36 \quad \beta_{HS}(2.5) = 0.15 \quad (30)$$

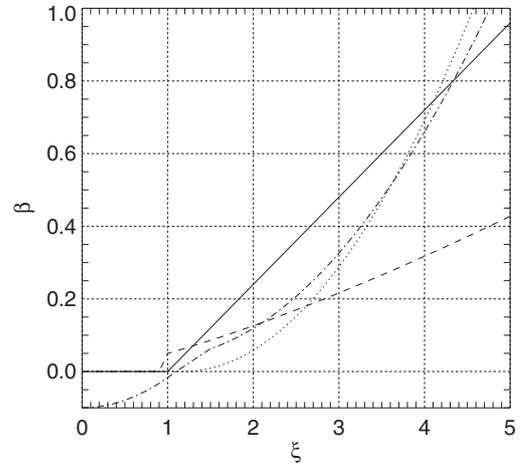


Fig. 1. Four cases of function $\beta(\xi)$ along the wind ($\theta = 0$) used in the numerical experiments. Solid line: Snyder-Hasselmann-Bosenberg case Eq. (26), dashed line: Hsiao-Shemdin case Eq. (27), dashed-dotted line: Chalikov case (Chalikov, 1995) and dotted line: ZRP case Eq. (28).

This serious difference is explained by lack of accuracy in both experiments (see Tolman and Chalikov (1996)). Fig. 6 of paper Snyder et al. (1981a) and Fig. 4 of the paper Hsiao and Shemdin (1983) show that the experimental data scatter has the same order as the mean values. Thus, the offered forms of the source functions Eqs. (26), (27) are not seriously justified. However, Hsiao-Shemdin data appears to be more trustworthy. It seems quite obvious that the Snyder function overestimates the wind input, by several times. Presented numerical experiments justify this conjecture.

For the ZRP function

$$\beta_{ZRP}(1.5) = 0.086 \quad \beta_{ZRP}(2.5) = 0.17 \quad (31)$$

and in the interval $1 < \xi < 3$

$$\beta_{SHB}(\xi) < \beta_{ZRP}(\xi) < \beta_S(\xi) \quad (32)$$

Fig. 1 presents one-dimensional plots of four functions, $\beta(\xi)$, studied in numerical experiments presented below. We intentionally did not include the description of the sophisticated Chalikov algorithm (Chalikov, 1995) for corresponding $\beta(\xi)$, for the sake of space.

We can conclude that, at the moment, there is no reliable parameterization of S_{in} , accepted by the entire oceanographic community. Keeping that fact in a mind, we decided to go our own way; this way is not focused on building new theoretical models, nor will it reconsider measurements of S_{in} .

For 68 years, since the works of Sverdrup and Munk (Sverdrup and Munk, 1947), physical oceanography assimilated tremendous amount of wind-wave experimental data - wave energy and spectral peak frequency- as functions of limited fetch. Such experiments are analyzed in works of Badulin et al. (2005); 2007); Gagnaire-Renou et al. (2011); Zakharov et al. (2014). On the other hand, numerical methods for solving the HE Eq. (1), with exact S_{nl} terms, have been improved significantly, for duration-limited and fetch-limited domains, as well.

Therefore, a new, purely pragmatic, approach for defining S_{in} was proposed. The function S_{in} has been chosen in such a way, so that numerical solutions of the Hasselmann equation explain the most known field experiments. The result was the S_{in} function, described in detail in Zakharov et al. (2012), and named thereafter: the ZRP function.

It is important to emphasize that work Zakharov et al. (2012) assumed localization of energy dissipation, in short waves. This assumption contradicts widely accepted concepts, but we explain the differences in the following section.

3. Two scenarios of wave-breaking dissipation term: spectral peak or high-frequency domination?

In this section, we explain why there is no need to use dissipation, in the spectral peak area.

Spectral peak frequency damping is a widely accepted practice, and is included as an option in the operational models WAM, SWAN and WaveWatchIII. Notice that different operational models use completely different long-wave dissipation functions.

The form of WAM3 spectral peak dissipation used in this paper is given by the following definition (Tolman, 2013) (the original notations are preserved):

$$S_{ds}(k, \theta) = C_{ds} \hat{\sigma} \frac{k}{\hat{\alpha}} \left(\frac{\hat{\alpha}}{\hat{\alpha}_{PM}} \right)^2 N(k, \theta) \quad (33)$$

$$\hat{\sigma} = (\overline{\sigma^{-1}})^{-1}$$

$$\hat{\alpha} = E \hat{\sigma}^4 g^{-2}$$

where $N(k, \theta)$ is the wave action spectrum, σ is the frequency, k is the wavenumber, θ is the angle, E is the total energy, $C_{ds} = -2.36 \times 10^{-5}$, $\hat{\alpha}_{PM} = 3.02 \times 10^{-3}$ is the value of $\hat{\alpha}$ for PM spectrum. Overline notation in $(\overline{\sigma^{-1}})^{-1}$ means averaging over the spectrum.

This formula implicitly assumes that dissipation is concentrated in the long-wave region, and numerical experiments, below, show that it is indeed realized that way, see Fig. 23.

It is important to emphasize that Eq. (33) is not supported by laboratory or field experiments, nor by analytical theory, nor numerical simulations, in the framework of phase-resolving numerical models. This is a heuristic construction, and it is important to trace its origin. Eq. (33) appeared in the paper of Komen et al. (Komen et al., 1984), and exerted strong influences on future developments of physical oceanography.

The authors of article (Komen et al., 1984) analyzed the energy balance, in the surface of the wind-driven sea, and concluded that the introduction of artificial dissipation term Eqs. (33) is necessary for explanation of experimental facts. This analysis was unsatisfactory for two reasons.

The authors of Komen et al. (1984) considered that existence of the "fully developed" sea, which is not only stationary in time, but homogeneous in space, as an obvious fact. For this reason they neglected the advection term $C_g \frac{\partial \varepsilon}{\partial x}$ in their analysis. In fact, all known stationary spectra vary with the fetch. It makes the concept of the "fully developed sea" doubtful.

Another weak point of the paper (Komen et al., 1984) is its uncritical use of the Snyder source function. As was shown before, this function has shaky foundations. Our numerical calculations show that it overestimates the energy growth rate by a factor of 5–6. The authors of Eq. (33) were using the Hsiao-Shemdin source function, which would hardly support Eq. (33) dissipation function, seeing in this case the balance had to be shifted to the dissipation side.

Anyways, for more than 30 years the dissipation term Eq. (33), together with Snyder input term, dominated in the operating models. These choices became a sort of credo, in physical oceanography. The purpose of the presented paper is its revision.

The dissipation of water surface waves, due to white-capping, is an extremely important physical phenomenon, not yet properly studied. Longuet-Higgins spent a lot of effort to develop an analytical theory of the wave-breaking (Longuet-Higgins, 1980b; 1980a; 1981; 1982). He found the set of interesting exact and approximate solutions of the Euler equations, describing potential flow, of ideal fluid with the free surface, but didn't solve the problem completely.

The difficulty of development in wave-breaking analytical theory is explained by sophisticated mathematical reasoning. Most probably, the system of Euler equations, for incompressible ideal fluid potential flow, with free surface on deep water in 1 + 1 geometry (i.e. depth coordinate and one horizontal coordinate) is the completely integrable system. It has too many peculiar features: cancellation of non-trivial four-wave interactions (Dyachenko and Zakharov, 1994), presence of an indefinite number of extra motion constants (Zakharov and Dyachenko, 2012), partial solutions describing propagating capillary waves, expressed in elementary functions (the "Crappier solution" (Crappier, 1957)). So far, the exact integrability, for the general time-dependent problem, is established in the exotic case of "asymptotically upwelling flow", in the absence of gravity (Zakharov, 2015b). An infinite number of exact solutions were found in the paper. Some of those solutions were published long ago by Longuet-Higgins (Longuet-Higgins, 1982).

Integrability makes the theory of white-capping complicated for the following reason. Integrability means absence of a universal scenario, of this effect. From the view-point of general nonlinear wave dynamics, wave-breaking is an example of "weak-collapse" (Zakharov and Kuznetsov, 2012). Such collapses are described, as a rule, by self-similar solutions.

Breakers, described by self-similar solutions

$$\eta(x, t) = g(t_0 - t)^2 F\left(\frac{x - x_0}{g(t_0 - t)^2}\right) \quad (34)$$

were studied analytically and numerically, in the framework of the simplified (and non-integrable!) MMT (Maida-McLaughlin-Tabak) model of Euler equations (Pushkarev and Zakharov, 2013b). Here $\eta(x, t)$ is the water surface elevation, x and t are spatial coordinate and time, respectively, and F is self-similar function.

Solution Eq. (34) describes formation of the wedge, with the top at $x = x_0$, at the moment of time $t = t_0$. Exact Euler equations have similar solutions, describing formation of locally stationary "corner flow solution", with angle 120° (see Longuet-Higgins (Longuet-Higgins, 1980a)). But in the MMT model, the non-integrable case, the self-similar breaker is a "global attractor". In other words, all breakers are self-similar. The exact Euler equation has the same self-similar solution describing formation of the "Stokes corner flow" (Longuet-Higgins, 1980a), but now it is not a generic scenario. In the general case, formation of the wedge is only the first stage of the breaker evolution. Later it ejects an inclined "Dirichlet jet" (Longuet-Higgins, 1980b), which plunges back into the water and transforms mechanical energy to heat. This scenario was qualitatively described in an article of Longuet-Higgins, and is supported by many numerical simulations (Banner and X.Tian, 1998)–Song and Banner (2002), laboratory experiments (Bonmarin, 1989), and field observations (Song and Banner, 2002)–(Kleiss and Melville, 2002). The literature on this subject is huge, and only a small portion of it is cited.

In spite of the complexity of this scenario, in terms of Fourier transform, the physical picture of the phenomenon is, more or less, universal. On the stage of wedge formation, the spatial Fourier spectrum, of energy, forms a "fat tail". Up to a certain moment of time, this spectrum is reversible in time. Plunging of the jets causes formation of the drops and bubbles, leading to dissipation of the energy and irreversibility. This is the mechanism of "high-frequency dissipation". The presence of high-frequency dissipation "chops off" the end of the tail, and violates the tail invertibility. Low and high harmonics, however, are strongly coupled in this event, due to strong, nonlinear, non-local, interaction, and deformed high wave-numbers, so the tail, almost immediately, returns to the spectral peak area. As soon as the fat spectral tail returns to the spectral peak area, total energy in the spectrum diminishes, causing settling of the spectral peak at a lower level of energy. This process of "shooting" of the spectral tail toward high

wave-numbers, and its returning back, due to wave breaking, is the real reason of “sagging down” of the energy profile in the spectral peak area. This was erroneously associated with the presence of the damping in the spectral peak area. This explanation suggests that individual wave-breaking studies (Young and Babanin, 2006; Tsagareli et al., 2009) do not prove the presence of spectral peak damping.

There is another question of fundamental importance. What is the speed, C_b , of breaker propagation? This C_b is connected with the characteristic length of a breaker by $P_b \simeq \frac{C_b^2}{g}$. The breaker propagation speed is the subject of direct measurements. Breakers produce strips of foam, and propagation of these strips can be traced relatively easily. The results of numerous experiments performed by Huang et al. (Hwang, 2007; Hwang and Wang, 2004; Hwang et al., 2013; Huang et al., 2013), Gemmrich et al. Gemmrich et al. (2008) gave, approximately, the same result: most of the breakers are “slow”. Their propagation speed is $C_b \simeq 0.2C_p$. Slow breakers are quasi-one-dimensional, but they propagate in a broad sector of angles with respect to the wind. Some breakers are fast ($C_b \simeq C_p$). Fast breakers propagate in the same direction as the leading wave. What is important is that “slow” and “fast” breakers are formed, but for completely different reasons. Let us look at the KZ spectrum

$$\varepsilon_\omega \simeq \omega^{-4}, \quad I_k \simeq k^{-5/2} \quad (35)$$

This spectrum is concentrated on fractals, non-smooth functions, and cannot be extended very far in the high frequency. At $\omega \simeq 3\omega_p$ the KZ spectrum turns into the Phillips spectrum $\varepsilon \simeq \omega^{-5}$, $I_k \simeq k^{-3}$, which is concentrated on wedge-like functions. Thus, formation of “slow breakers” is an unavoidable consequence of the energy flux to high frequency region, provided by four-wave, nonlinear, wave interaction. Thus, it is reasonable to suppose that wave-breaking dissipation is localized in short scales.

The population of waves having frequencies 3–4 times bigger than spectral maximum frequency is called “Phillips sea” (Phillips, 1966; 1985; Newell and Zakharov, 2008), which we call for the sake of brevity the “short waves”. The “Phillips sea” contains no more than 2% of the total wave energy, but the whole energy dissipation, fueled by energy flux from long waves, is happening right there. It is proved, experimentally, that “Phillips sea” is described by a universal Phillips spectrum $\varepsilon \simeq \frac{\alpha g^2}{\omega^5}$, where $\alpha \simeq 0.01$ is a dimensionless constant, while for the Pierson–Moscowitz spectrum $\alpha = 0.0081$ (Phillips, 1966; 1985; Newell and Zakharov, 2008).

“Phillips sea” is quite an interesting physical object. It contains breakers of different, statistically uniformly distributed, sizes (Newell and Zakharov, 2008), down to characteristic wave length $\lambda \simeq 3\text{--}5$ cm, where capillary effects become essential.

The exact form of the “Phillips sea” energy dissipation function is unknown. Recently, a quite plausible model of such function has been presented in Badulin and Zakharov (2012), which is hoped to become the subject of oceanographic community discussion.

What’s about the “fast breakers”? They rarely appear in 1 + 1 geometry, where nontrivial four-wave interactions are canceled out, and there is no energy flux to high wavenumbers.

The steepness, in the conditions of typically developed wave turbulence, is not big: $\mu = \langle \nabla \eta^2 \rangle^{1/2} \sim 0.1$, or even smaller. Because this value is very far from limiting steepness of Stokes wave $\mu \simeq 0.3$, these waves are, essentially, weakly-nonlinear. Besides those waves, shorter waves inevitably develop, having steepness approaching the critical one, and those waves break.

However, there is also another process- the modulational instability or “wave grouping”- which leads to spatial inhomogeneity and formation of “rough waves” propagating, with the spectral peak velocity. Theory of rogue wave formation is a separate, and interesting phenomenon, which is discussed in many articles

(see, for instance Gemmrich and Garrett (2008); Zakharov and Dyachenko (2008); Dyachenko and Zakharov (2005)), but is not the subject of the current paper.

It is important that direct numerical solutions of both exact (Zakharov et al., 2009) and approximate (Dyachenko et al., 2015) primordial Euler equations show that dissipation of the rogue waves does not make any significant contribution into energy balance of wind-driven seas. Thus, the main conclusion about the dissipation taking place in short scales remains unchanged.

4. Numerical experiments set-up

The subject of numerical simulation was the stationary HE Eq. (10), for different wind input functions. A total of 5 different wind inputs have been tested in the frame of stationary HE:

$$\frac{\partial \omega_k}{\partial \bar{k}} \frac{\partial \varepsilon}{\partial \bar{r}} = S_{nl} + S_{in} + S_{diss} \quad (36)$$

The first 4 tests were done assuming the absence of low-frequency dissipation and the presence of “implicit” high-frequency dissipation.

All simulations were performed with the help of the WRT method (Tracy and Resio, 1982; Vledder, 2006; Resio and Perrie, 1991), previously used in Perrie and Zakharov (1999); Resio and Perrie (1989); Resio et al. (2004); Long and Resio (2007); Badulin and Zakharov (2012); Webb (1978); Korotkevich et al. (2008), on the grid of 71 points in frequency and 36 points in angle domains. A constant wind of speed 10 m/s is assumed to be blowing away from the shore line, along the fetch. The assumption of the constant wind speed is a necessary simplification, due to the fact that the numerical simulation is being compared to various data from field experiments, and the considered set-up is the simplest physical situation, which can be modeled.

4.1. The details of “implicit” damping implementation

One should specifically stop and note details of the “implicit” high-frequency damping, used in all five numerical simulations. Including the “implicit” damping consists in continuation of the spectral tail by Phillips law (Phillips, 1966) $A(\omega_0) \times \omega^{-5}$, where $A(\omega_0)$ is the parameter dynamically changing in time.

The coefficient $A(\omega_0)$, in front of ω^{-5} , is not exactly known, but is not required to be defined in explicit form - it is dynamically determined from the continuity condition of the spectrum, at frequency ω_0 , on every time step. In other words, the starting point of the Phillips spectrum coincides with the last point of the dynamically changing spectrum, at the frequency point $\omega_{crit} = 2\pi f_{crit}$, where $f_{crit} \simeq 1.1$ Hz, as per Resio and Long experimental observations (Long and Resio, 2007). This is the way the high frequency “implicit” damping is incorporated into the alternative computational framework of HE.

One should note recently developed analytical models, which automatically describe the transition from the KZ spectrum ω^{-4} to Phillips tail ω^{-5} (Badulin and Zakharov, 2012). Such modification of the “implicit” damping is in future plans, but the question of the finer details of the high-frequency “implicit” damping structure is of secondary importance, at the current “proof of the concept” stage, of the alternative framework development.

4.2. WTT facts used in numerical simulation

As a rule, confirmed by field and numerical observations, the wave energy spectrum has sufficiently sharp peak at $\omega \simeq \omega_p$. However, almost immediately after the spectral peak at $\omega \simeq 1.5\omega_p$ the

advection term $\frac{\partial \omega}{\partial k} \frac{\partial \varepsilon}{\partial r}$ becomes insignificant, and the original stationary HE Eq. (10) is transformed into

$$S_{nl} + S_{in} + S_{diss} = 0 \quad (37)$$

Comparison with Eq. (13) shows that Eq. (37) can be rewritten in the form

$$\varepsilon(\omega, \theta) = \frac{F(\omega, \theta)}{\Gamma(\omega, \theta) - \gamma(\omega, \theta) + \gamma_{diss}(\omega, \theta)} \quad (38)$$

As it was shown in Zakharov (2010); Zakharov and Badulin (2011), nonlinear dissipation $\Gamma(\omega, \theta)$ in the “universal area” $\omega > 1.5\omega_p$ is several times greater than wind forcing term. Therefore, Eq. (37) can be rewritten, as a first approximation, by

$$S_{nl}(\omega, \theta) = 0 \quad (39)$$

or

$$\varepsilon(\omega, \theta) = \frac{F}{\Gamma} \quad (40)$$

As was mentioned before, this equation has a rich family of solutions. The simplest and best known solution is the isotropic Zakharov-Filonenko solution (Zakharov and Filonenko, 1967)

$$\varepsilon(\omega, \theta) = \frac{C_p g^{4/3}}{\omega^4} P^{1/3} = \frac{\beta_{KZ}}{\omega^4} \quad (41)$$

Here P is the energy flux into the high wavenumbers region.

The energy density flux per square unit, in the atmosphere, is $P_{sq} = \rho_a U^3$, where ρ_a is the atmosphere density and U is the wind speed. For $U = 10$ m/s, $P_{sq} \approx 1.2$ kW. A relatively small part of this flux is transferred to the water. According to Hwang and co-authors (Hwang et al., 2013; Huang et al., 2013), the estimated amount, transferred to the ocean, is $P_0 \approx 0.1$ Wt. Approximately one third of this amount is spent into energy flux formation, toward high wave numbers. In “oceanographic” normalization this flux has to be divided by $\rho_w g$, where $\rho_w \approx 10^3$ kg/m³. Finally, the energy flux toward small scales is

$$P \sim 2 \div 3 \times 10^{-6} \text{ m}^2/\text{s} \quad (42)$$

This expression agrees with the presented numerical experiments. The Kolmogorov constant in the Eq. (41) can be found numerically (Zakharov, 2010). Recently, its value has been found more rigorously (Geogdjaev and Zakharov, 2015):

$$C_p = 4\pi \times 0.194 = 2.43 \quad (43)$$

One can estimate the characteristic value of β_{KZ} :

$$\beta_{KZ} = C_p g^{4/3} P^{1/3} \approx 0.6 \text{ m}^2/\text{sec}^3 \quad (44)$$

According to WTT, the value of β_{KZ} should be constant, somewhere in the region $1.5\omega_p < \omega < \omega_{crit}$. Here, $\omega_{crit} = 2\pi \times 1.1 = 6.91$ is the critical frequency at which the “implicit” damping is turning on. The energy flux, for this area, is diminishing proportionally to ω^{-3} , and β_{KZ} is not constant anymore - one has to substitute β_{KZ} by $\beta_{KZ} \cdot \frac{\omega_{crit}}{\omega}$.

The general, anisotropic KZ solution, with zero wave-action flux from $\omega \rightarrow \infty$, can be presented in the form (Zakharov, 2010)

$$\varepsilon(\omega, \theta) = \frac{\beta_{KZ}}{\omega^4} F\left(\frac{\omega_s}{\omega}, \theta\right) \quad (45)$$

where $\omega_s = \frac{M}{p}$, M - the momentum flux to the small scale region. The frequency ω_s depends on the shape of S_{in} , in a typical case $\omega_s \approx \omega_p$. $F\left(\frac{\omega_s}{\omega}, \theta\right)$ is a “structural function”. It is established that in the limit $\frac{\omega_s}{\omega} \rightarrow 0$ (see Katz et al. (Katz and Kontorovich, 1975), Zakharov (Zakharov, 2010)):

$$F\left(\frac{\omega_s}{\omega}\right) \rightarrow 1 + c_2 \frac{\omega_s}{\omega} \cos \theta \quad (46)$$

where c_2 is the “second Kolmogorov constant”.

If one uses, for S_{nl} , the “diffusion approximation” (Zakharov and Pushkarev, 1999), the structural function is known

$$F\left(\frac{\omega_s}{\omega}, \theta\right) = \left(1 + \frac{\omega_s}{\omega} \cos \theta\right)^{1/3} \quad (47)$$

As far as $F \rightarrow 1$ at $\omega \rightarrow \infty$, the KZ solution Eq. (45) describes well-known “angular spreading”. This solution becomes isotropic at $\omega \rightarrow \infty$.

One can introduce

$$f\left(\frac{\omega_s}{\omega}\right) = \frac{1}{2\pi} \int_0^{2\pi} F\left(\frac{\omega_s}{\omega}, \theta\right) d\theta \quad (48)$$

From Eq. (46), one can see that $f\left(\frac{\omega_s}{\omega}\right) \rightarrow +\lambda\left(\frac{\omega_s}{\omega}\right)^2 + \dots$

A more detailed study, of the functions $F\left(\frac{\omega_s}{\omega}, \theta\right)$ and $f\left(\frac{\omega_s}{\omega}\right)$, is an urgent theoretical problem, but is out of the scope of the current paper. One can expect, however, that $f\left(\frac{\omega_s}{\omega}\right)$, for $\omega > 2\omega_p$, is close to 1. Presented calculations confirm this conjecture: the compensated angle-averaged spectrum $\langle \varepsilon \rangle < \omega^4$ is constant, up to 20% accuracy inside, the spectral band $0.4\text{Hz} < f < 1\text{Hz}$.

Notice that the average energy spectrum, $\langle \varepsilon \rangle$, decays in this spectral band, by a factor of 40, and the difference of the KZ spectrum and Phillips spectrum is essential.

Let’s discuss self-similar solutions of the conservative HE Eq. (24). This equation has a family of self-similar solutions, which can be conveniently rewritten in the form

$$\varepsilon(\omega, \theta, \chi) = \chi^{p+q} F(\xi, \theta) \quad (49)$$

$$\xi = \omega \chi^q \quad (50)$$

where $F(\chi, \theta)$ is the function, satisfying the relation

$$\frac{\cos \theta}{2\xi} \left[(p+q)F + q\xi \frac{\partial F}{\partial \xi} \right] = S_{nl} \quad (51)$$

and q and p are the constants, connected by the “magic relation” Eq. (18). If the self-similar solution is realized, then dimensionless energy and frequency are the power functions, of the dimensionless fetch; see Eq. (16), (17).

It was shown in Zakharov et al. (2012) that self-similar solutions also exist for the case of wind input, if $\beta(\xi)$ in Eq. (22) is the power function of the frequency:

$$\beta = \omega^s f(\theta) \quad (52)$$

The constants p and q are defined now, unambiguously as

$$q = \frac{1}{2+s} \quad (53)$$

$$p = \frac{8-s}{2(2+s)} \quad (54)$$

As was already mentioned, practically all ocean field measurements demonstrate power dependencies Eq. (16), (17). However, there is scattering in the definition of the exponent p , in the range $0.74 < p < 1.1$. It’s quite possible that this scattering is due to absence of the universal expression for S_{in} , suitable for any atmospheric boundary layer state.

The experiments of Kahma (see Zakharov (2015a) for references), performed for “stable” and “unstable” atmosphere, gave different values of p . However, the “magic relation” Eq. (18) still holds true for those different cases. This fact holds the promise that WTT always works.

More than the half of the numerical experiments have the values $p = 1$, $q = 0.3$. Such self-similarity occurs if $s = 4/3$. This fact, together with field experimental data (Long and Resio, 2007), leads to the appearance of the ZRP wind input term (Zakharov et al., 2012).

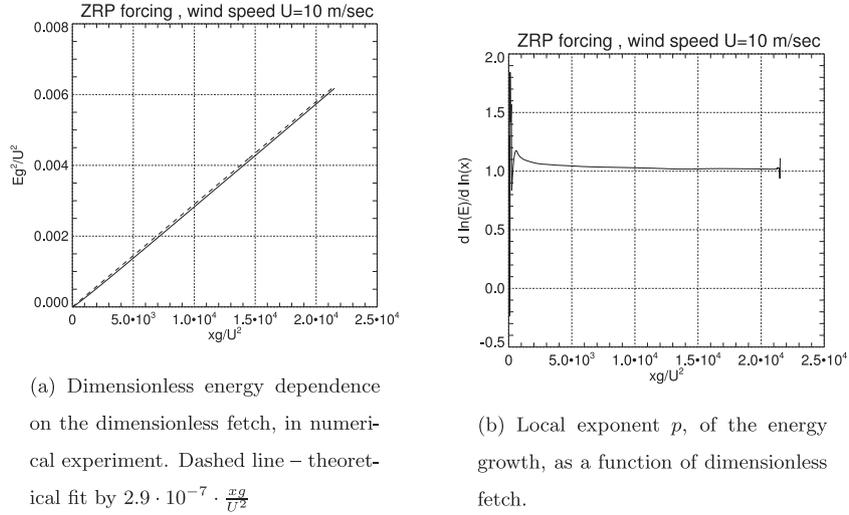


Fig. 2. (a) Dimensionless energy dependence on the dimensionless fetch, in numerical experiment. Dashed line – theoretical fit by $2.9 \cdot 10^{-7} \cdot \frac{xg}{U^2}$. (b) Local exponent p , of the energy growth, as a function of dimensionless fetch.

5. Numerical study of different wind input models

The current section presents the results of numerical simulation, of different wind input models, in the alternative framework of *HE*, for limited fetch statement, based on the following:

1. Exact S_{nl} term
2. Absence of spectral maximum dissipation (excluding special *WAM3* case)
3. High-frequency “implicit” dissipation

The first numerically studied wind input model is described in the previous section’s *ZRP* model:

$$S_{in}(\omega, \phi) = \gamma(\omega, \phi) \cdot \varepsilon(\omega, \phi) \quad (55)$$

$$\gamma = 0.05 \frac{\rho_{air}}{\rho_{water}} \omega \left(\frac{\omega}{\omega_0} \right)^{4/3} f(\theta) \quad (56)$$

$$f(\theta) = \begin{cases} \cos^2 \theta & \text{for } -\pi/2 \leq \theta \leq \pi/2 \\ 0 & \text{otherwise} \end{cases} \quad (57)$$

$$\omega_0 = \frac{g}{u_{10}}, \quad \frac{\rho_{air}}{\rho_{water}} = 1.3 \times 10^{-3} \quad (58)$$

Fig. 2 a shows total energy growing along the fetch, by the power law

$$\hat{\varepsilon} = \varepsilon_0 \chi$$

in accordance with Eq. (16) for $p = 1.0$, see Fig. 2b.

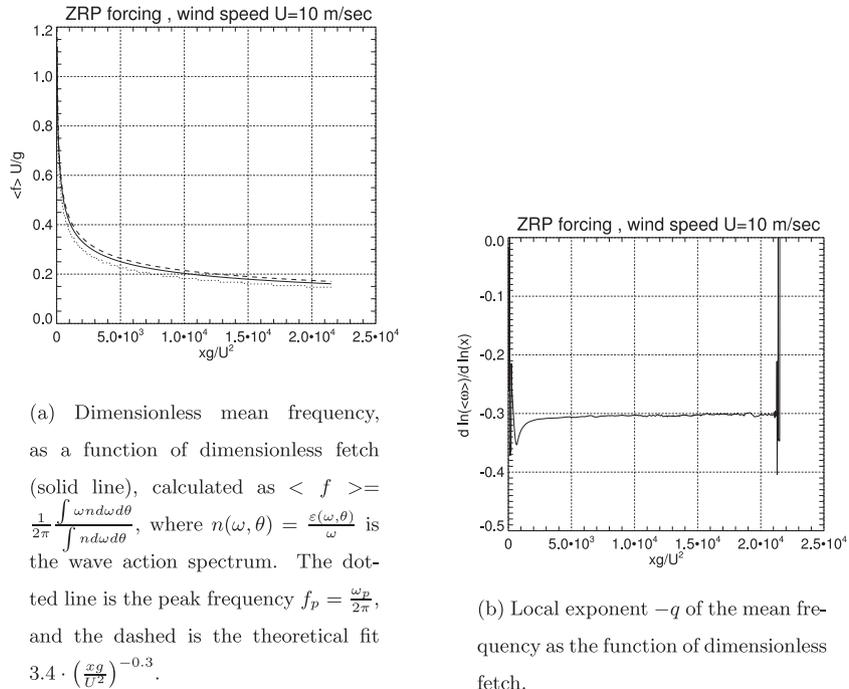


Fig. 3. (a) Dimensionless mean frequency, as a function of dimensionless fetch (solid line), calculated as $\langle f \rangle = \frac{1}{2\pi} \frac{\int \omega n d\omega d\theta}{\int n d\omega d\theta}$, where $n(\omega, \theta) = \frac{\varepsilon(\omega, \theta)}{\omega}$ is the wave action spectrum. The dotted line is the peak frequency $f_p = \frac{\omega_p}{2\pi}$, and the dashed is the theoretical fit $3.4 \cdot \left(\frac{xg}{U^2} \right)^{-0.3}$. (b) Local exponent $-q$ of the mean frequency as the function of dimensionless fetch.

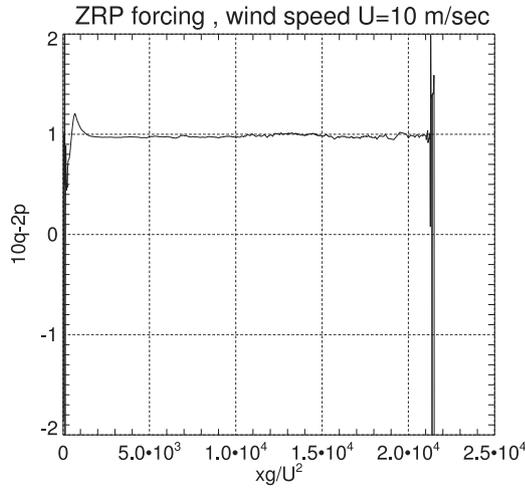


Fig. 4. “Magic relation” $(10q - 2p)$ as the function of dimensionless fetch for ZRP wind input term.

Table 1
Exponents p and q (see Eqs. (16),(17)) for 12 different experimental observations (Badulin et al., 2007) with the last row corresponding to limited fetch growth numerical experiment within alternative ZRP framework.

Experiment	p	q	$C = 10q - 2p$
Babanin, Soloviev 1998 Black Sea	0.89	0.28	1.02
Walsh et al. (1989) US coast	1.0	0.29	0.90
Kahma, Calkoen (1992) unstable	0.94	0.28	0.92
Kahma, Pettersson (1994)	0.93	0.28	0.94
JONSWAP by Davidan (1980)	1.0	0.28	0.80
JONSWAP by Phillips (1977)	1.0	0.25	0.75
Kahma, Calkoen (1992) composite	0.9	0.27	0.90
Kahma (1981, 1986) rapid growth	1.0	0.33	1.03
Kahma (1986) average growth	1.0	0.33	1.03
Donelan et al.(1992) St Claire	1.0	0.33	1.03
JONSWAP by Hasselmann et al. (1973)	1.0	0.33	1.03
Mitsuyasu et al. (1971)	1.0	0.33	1.03
ZRP numerics	1.0	0.3	1.00

Dependence of mean frequency, on the fetch, shown in Fig. 3a, demonstrates the law

$$\hat{\omega} = \omega_0 \chi^{-0.3}$$

in good correspondence with self-similar dependence Eq. (17), for $q = 0.3$, see Fig. 3b.

Fig. 3 a presents, not only mean frequency, but also the maximum spectral frequency. Their difference, however, is so small, that we will not distinguish between them, hereafter.

The result of the “magic relation” check is presented in Fig. 4. It presents the relation as a function of the fetch. It strongly agrees with the self-similar prediction of Eq. (18).

Table 1 presents the results (Badulin et al., 2007) of calculating the exponents p and q (see Eqs. (16), (17)), for 12 different experimental observations, with the last row corresponding to a limited fetch growth numerical experiment, within the alternative ZRP framework. The value of $C = 10q - 2p$, averaged over the experiments, is $\langle C \rangle = 0.95$. One can see correspondence with the predicted, theoretical, value $C_t = 1$, as well as the numerical result. One should note interpretations of the JONSWAP experiment, by different experts, provided different values of p and q , and, correspondingly, C .

Let us proceed with the analysis of numerical spectra. Typical, angle averaged, wind input function density $\langle S_{in} \rangle$ and angle averaged spectrum, in linear coordinates, are presented on Fig. 5. It is seen that a major portion of the wind forcing is concentrated in the spectral peak vicinity.

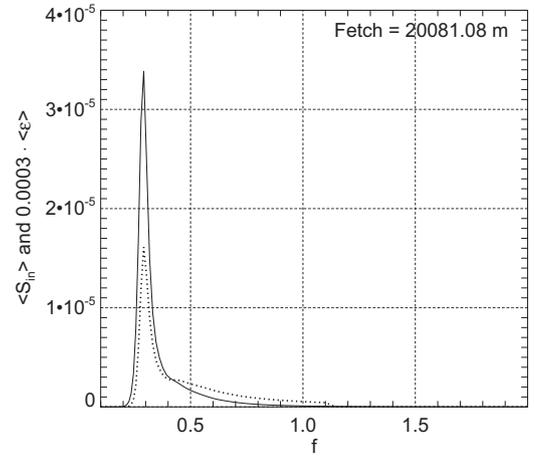


Fig. 5. Typical, angle averaged, wind input function density $\langle S_{in} \rangle = \frac{1}{2\pi} \int \gamma(\omega, \theta) \varepsilon(\omega, \theta) d\theta$ and angle averaged spectrum $\langle \varepsilon \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) d\theta$ (solid line) as the functions of the frequency $f = \frac{\omega}{2\pi}$.

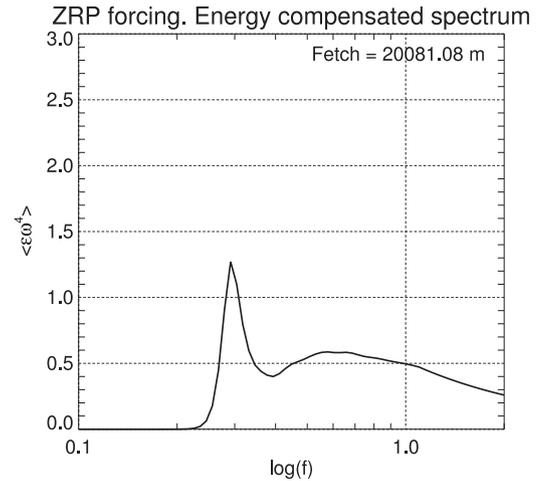


Fig. 6. Angle averaged, energy compensated, spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of frequency $f = \frac{\omega}{2\pi}$ shows the behavior close to the theoretically predicted value $\beta \approx C_K(g^2 P)^{1/3} \approx 0.5$ in the area of the “plateau”.

For the sake of brevity, the calculation of density flux to high wavenumbers is omitted, and only the final result is presented: $P \approx 2 \times 10^{-6} \text{ m}^2/\text{sec}$, which gives the value $\beta_{KZ} \approx 0.5 \text{ m}^2/\text{sec}^3$. An approximation of β_{KZ} is given by angle averaged compensated spectrum $\varepsilon \omega^{-4}$, shown in Fig. 6.

Finally, Fig. 7 presents angle averaged energy spectrum, as the function of frequency, in logarithmic coordinates. One can see that it consists of segments of:

1. Spectral maximum area
2. Kolmogorov–Zakharov spectrum ω^{-4}
3. Phillips high frequency tail ω^{-5}

It follows from Figs. 6 and 7 that, in the interval $0.4 \text{ Hz} < f < 1 \text{ Hz}$, the energy spectrum is close to the Zakharov-Filonenko spectrum Eq. (41), with the accuracy 20%. One cannot expect higher accuracy, due to the anisotropy of the realized spectrum and the influence of the high frequency dissipation, as well. One can state, nevertheless, that in the domain of frequencies exceeding spectral maximum frequencies, the energy spectrum is fairly close to the one described by equation $S_{nl} = 0$, which confirms the view that the energy balance of wind excited surface waves, as presented in current research.

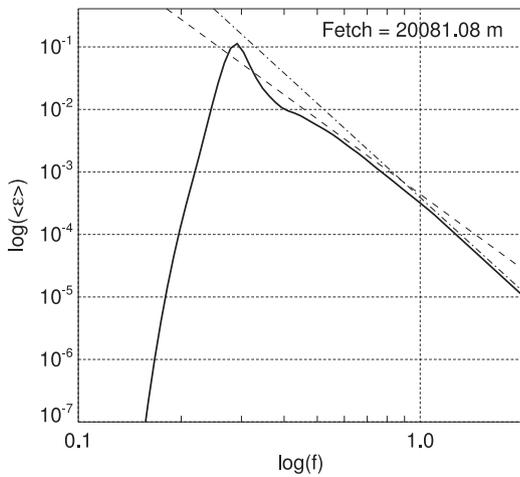


Fig. 7. Decimal logarithm of angle averaged spectral energy density $\langle \varepsilon \rangle = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon d\theta$, as a function of the decimal logarithm of the frequency $f = \frac{\omega}{2\pi}$ - solid line. Dashed line - fit $\sim \omega^{-4}$, dash-dotted line - fit $\sim \omega^{-5}$.

The analysis carried out in the previous section shows that the quality of the different versions of wind input terms S_{in} should be estimated by the following criteria:

1. Checking powers of the observed energy and mean frequency dependencies Eq. (16), (17) along the fetch, versus what is predicted by self-similar solutions.
2. Checking the “magic relations” Eq. (18) between exponents p and q , for observed energy and frequency dependencies, along the fetch.
3. Checking the exponents of directional (angle averaged) spectral energy dependencies versus the Kolmogorov–Zakharov exponent -4 .

We applied such tests to the results of HE simulations, which used the following popular wind input terms, within alternative framework:

1. Chalikov S_{in} term (Chalikov, 1995; Tolman and Chalikov, 1996)
2. Snyder S_{in} term (Snyder et al., 1981a)
3. Hsiao–Shemdin S_{in} term (Hsiao and Shemdin, 1983)
4. WAM3 S_{in} term (Tolman, 2013)

6. Test of Chalikov wind input term

The sophisticated Chalikov wind input term algorithm is not presented in the current paper, due to space. Curious readers can find it in Tolman and Chalikov (1996); Chalikov (1995).

Fig. 8 a shows that total energy growth, along the fetch, significantly exceeds what is observed in ZRP simulation. This

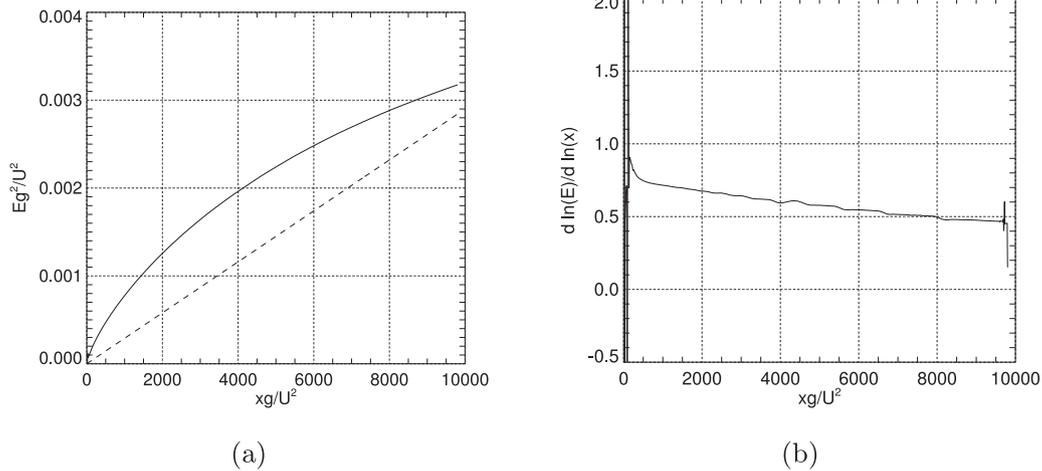


Fig. 8. Same as Fig. 2, but for Chalikov S_{in} .

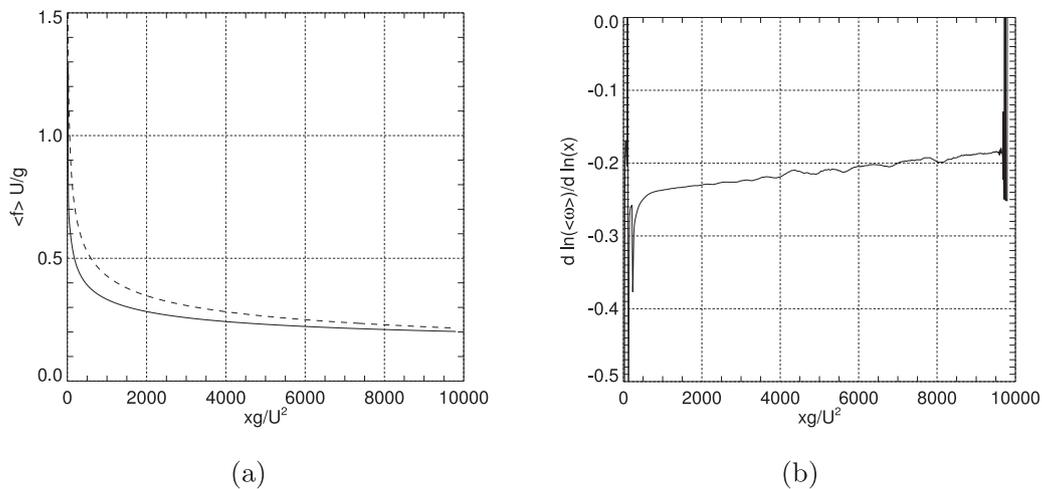


Fig. 9. Same as Fig. 3, but for Chalikov S_{in} .

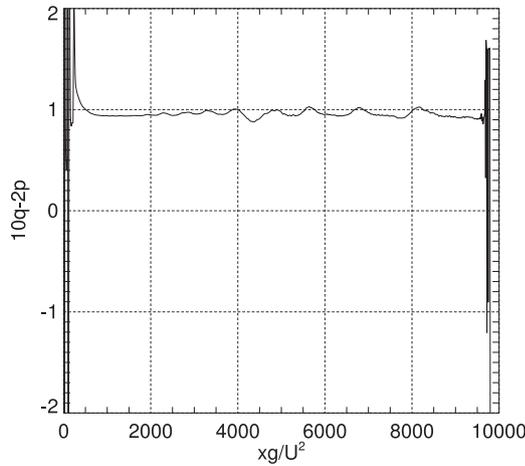


Fig. 10. “Magic relation” $(10q - 2p)$ as a function of the dimensionless fetch for Chalikov wind input term.

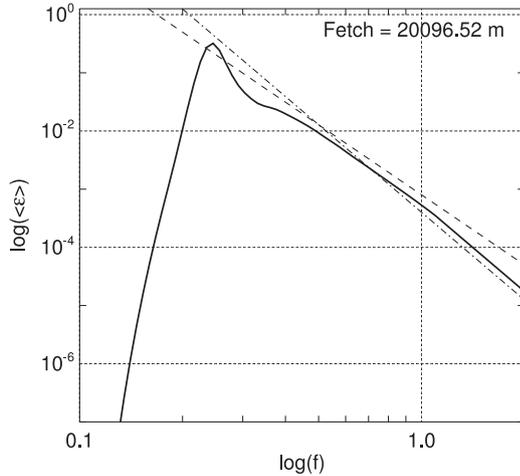


Fig. 11. Same as Fig. 7, but for Chalikov S_{in} .

dependence is not the power function of the fetch, see Fig. 8b, but can be approximated by relatively slowly changing values of the exponent p , ranging from 0.8 to 0.5, along the fetch. While $p \approx 0.8$ is still observed in some experiments, the value $p \approx 0.5$ is completely unrealistic.

The same relates to the mean frequency dependence, against the fetch, shown in Fig. 9a, with the values of the exponent q shown in Fig. 9b. The value of q is also not constant, but slowly diminishes with the fetch. One should note that the value $q \approx 0.25$ has been detected in the experiments, while $q < 0.2$ has never, apparently, occurred. Fig. 10 presents the combination, $(10q - 2p)$, as a function of the fetch. It is surprising that it is in good accordance with the relation Eq. (18). It means that despite incorrect values p and q along the fetch, their combination $(10q - 2p)$ still holds in complete accordance with theoretical prediction, and the spectra are “locally self-similar”, in accordance with WTT.

Fig. 11 presents directional (angle averaged) spectrum, as a function of frequency, in logarithmic coordinates. One can see Kolmogorov–Zakharov $\sim \omega^{-4}$ and Phillips high frequency tail $\sim \omega^{-5}$, as well.

Fig. 12 confirms the presence of $\sim \omega^{-4}$ spectrum, through existence of the “plateau” section, to the right of the spectral peak area, in the frequency range $0.45\text{Hz} < f < 1\text{Hz}$.

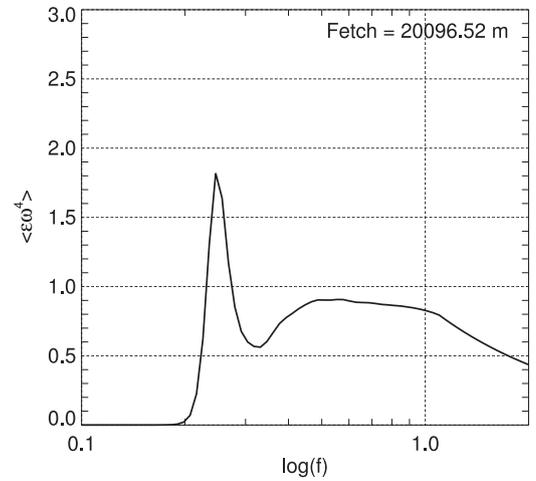


Fig. 12. Angle averaged energy compensated spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of frequency $f = \frac{\omega}{2\pi}$ for Chalikov S_{in} .

7. Test of Snyder wind input term

Snyder wind input term Eq. (26) is especially important, since it is included, as an option, in operational models.

The main disadvantage of the Snyder wind input term is rapid energy growth, with the fetch, presented in Fig. 13a. For dimensionless fetches $\chi \approx 5 \times 10^3$ it shows values approximately three times bigger than those experimentally observed and those obtained in the more realistic ZRP model. Apparently, that fact, together with a non-critical belief in the Snyder wind input function, caused the myth about long-wave dissipation, due to breaking of the long waves.

Despite unrealistic energy growth along the fetch, Fig. 13b shows that energy growth is close to the power function Eq. (16), with the index p slowly changing from 1 to 0.6, which is, generally, significantly lower than observed in field experiments.

The same relates to the mean frequency dependence, against the fetch, shown in Fig. 14a, with the values of the exponent q shown in Fig. 14b. The value of q is not constant, but slowly diminishes with the fetch. One should note that the value $q \approx 0.25$ has been detected in experiments, while $q < 0.2$ has never, apparently, occurred.

Dependence of the mean frequency, against the fetch, shown in Fig. 14a, is lower than ZRP numerical results, but can be also approximated by a power function of the fetch Eq. (17), with the values of q slowly diminishing along the fetch, from 0.3 to 0.25, see Fig. 14b.

The Kolmogorov–Zakharov spectrum $\sim \omega^{-4}$ and Phillips high frequency tail $\sim \omega^{-5}$ can be seen in Fig. 15, presenting directional spectrum as a function of frequency, in logarithmic coordinates. The span of the Kolmogorov–Zakharov $\sim \omega^{-4}$ segment can be estimated from Fig. 16.

Fig. 17 presents the combination $(10q - 2p)$ as a function of the fetch. Again, it strongly agrees with the theoretical relation Eq. (18). As in the Chalikov case, it means that, despite imperfect values of p and q and rapid energy growth along the fetch, their combination $(10q - 2p)$ still holds in complete accordance with theoretical prediction, i.e. self-similarity is also fulfilled, locally, in the Snyder case.

8. Test of Hsiao–Shemdin wind input term

Fig. 18a shows that total energy growth, along the fetch, underestimates ZRP simulation. It obeys the power law Eq. (16) with the exponent $p \approx 0.5$, see Fig. 18b.

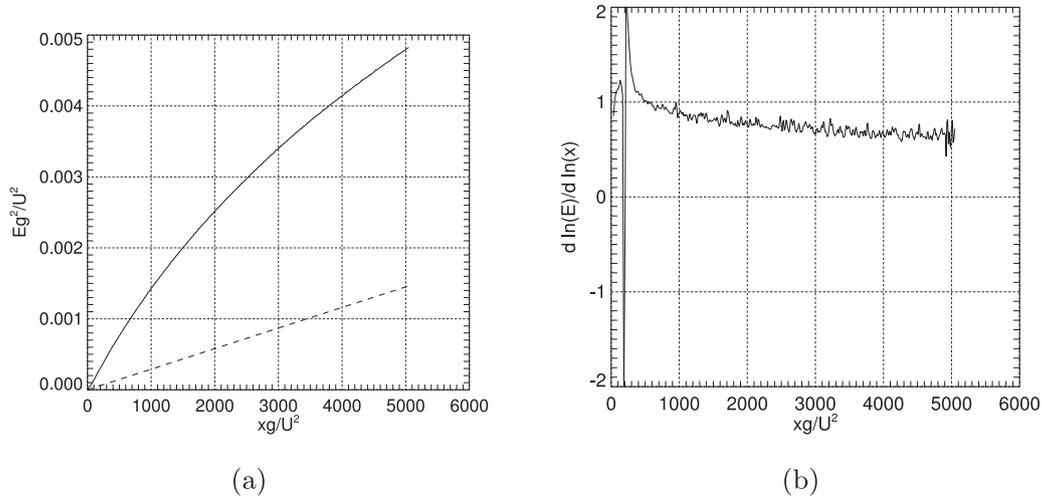


Fig. 13. Same as Fig. 2, but for Snyder S_{in} .

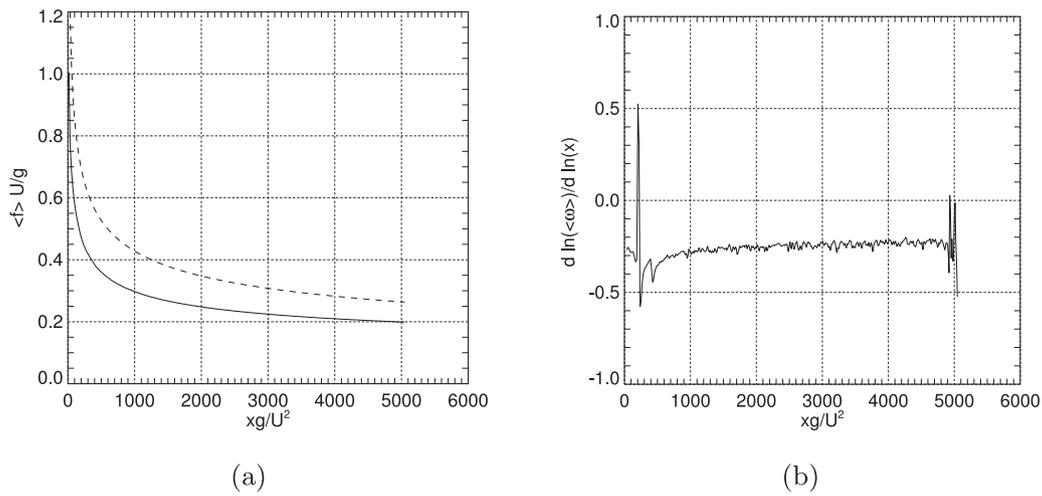


Fig. 14. Same as Fig. 3, but for Snyder S_{in} .

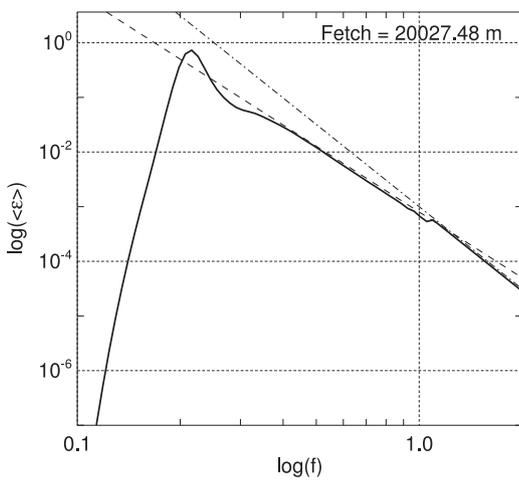


Fig. 15. Same as Fig. 7, but for Snyder S_{in} .

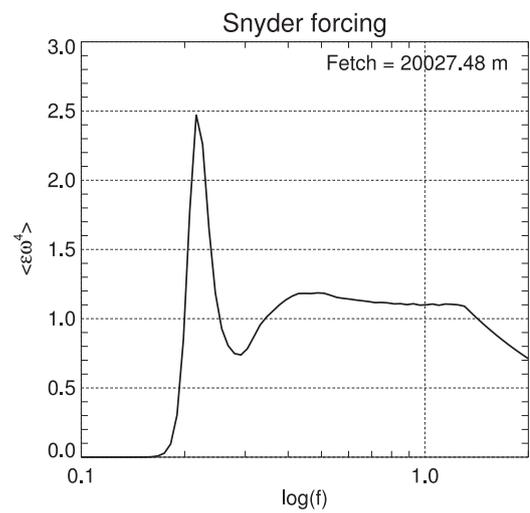


Fig. 16. Angle averaged, energy compensated, spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of the frequency $f = \frac{\omega}{2\pi}$, for Snyder S_{in} .

Fig. 19a demonstrates mean frequency dependence, on the fetch, by power law Eq. (17), with asymptotic value of the index $q \approx 0.21$, see Fig. 19b. Kolmogorov–Zakharov segment $\sim \omega^{-4}$ and Phillips high frequency tail $\sim \omega^{-5}$ can be seen in Fig. 20, presenting directional spectrum as a function of frequency, in logarithmic

coordinates. The span of the Kolmogorov–Zakharov spectrum can be estimated using Fig. 21.

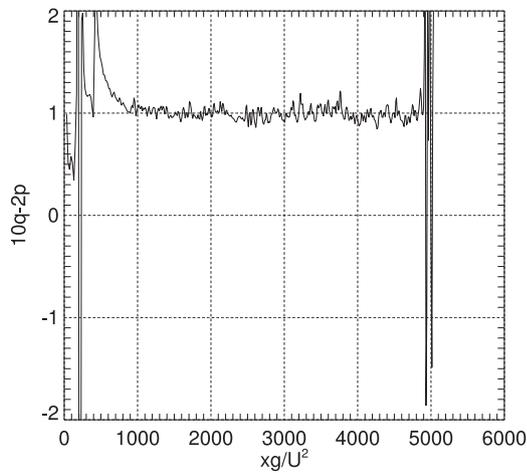


Fig. 17. Relation $(10q - 2p)$ as a function of the fetch x for Snyder wind input term.

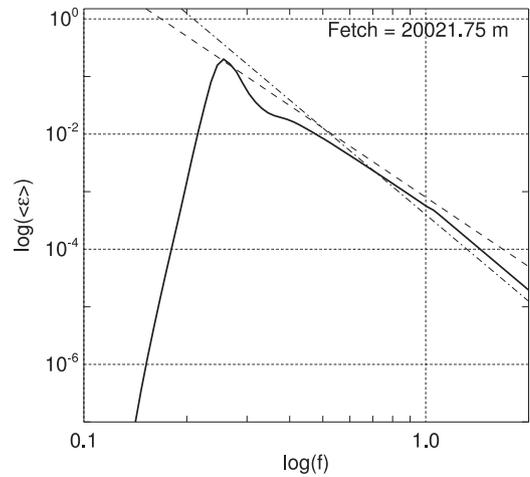


Fig. 20. Same as Fig. 7, but for Hsiao–Shemdin S_{in} .

Fig. 22 presents combination $(10q - 2p)$ as the function of the fetch. It is in total agreement with the theoretical predictions Eq. (18), which means that self-similarity is also fulfilled locally in Hsiao–Shemdin case.

9. Test of WAM3 input terms

The current section presents what is, arguably, the most important part of the paper. It concerns the numerical simulation performed in the frame of WAM3 model (Komen et al., 1984; Snyder et al., 1981b; Tolman, 2013), using exact expression for S_{nl} term. Similar experiments were performed by different authors, more

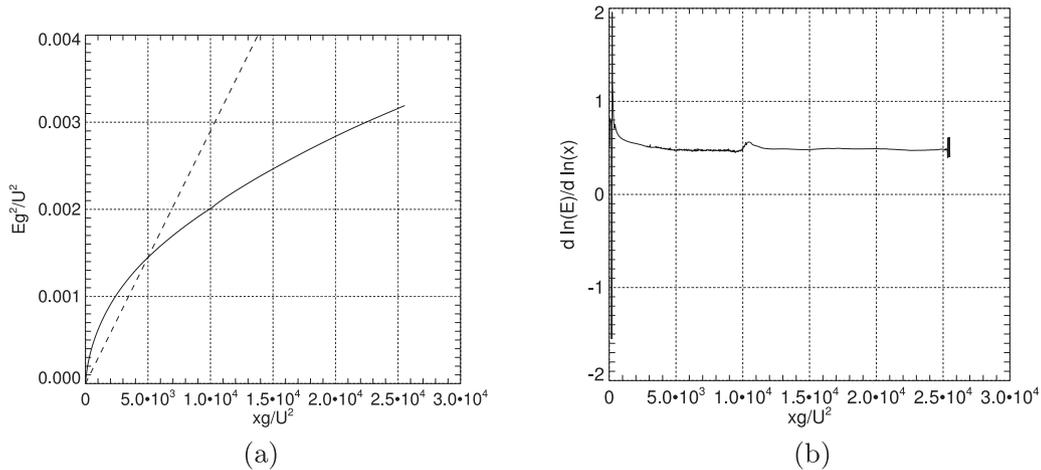


Fig. 18. Same as Fig. 2, but for Hsiao–Shemdin S_{in} .

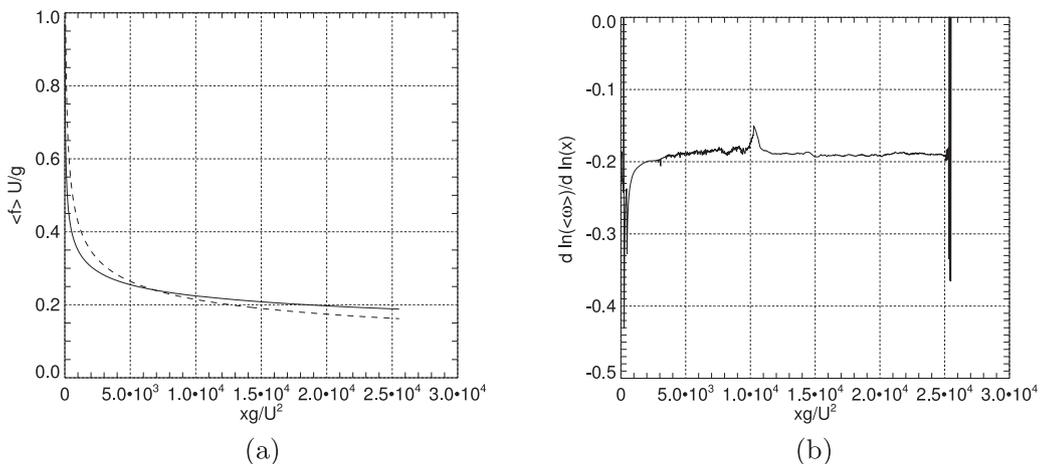


Fig. 19. Same as Fig. 3, but for Hsiao–Shemdin S_{in} .

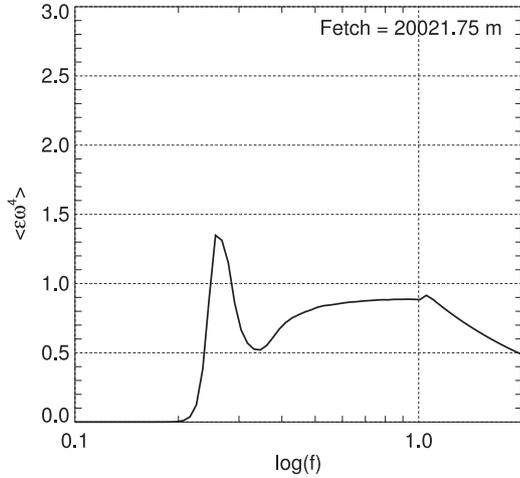


Fig. 21. Angle averaged, energy compensated, spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as the function of decimal logarithm of the frequency $f = \frac{\omega}{2\pi}$ for Hsiao-Shemdin S_{in} .

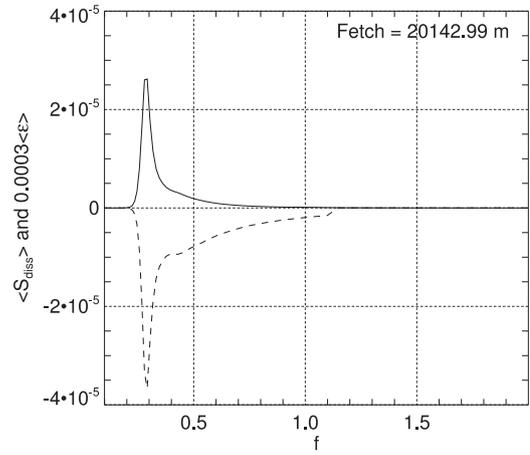


Fig. 23. Angle averaged energy dissipation function $\langle S_{diss} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \gamma_{diss} \varepsilon(\omega, \theta) d\theta$ (dotted line) and angle averaged scaled energy spectrum $\langle \varepsilon \rangle = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon(\omega, \theta) d\theta$ (solid line) as the functions of the frequency $f = \frac{\omega}{2\pi}$.

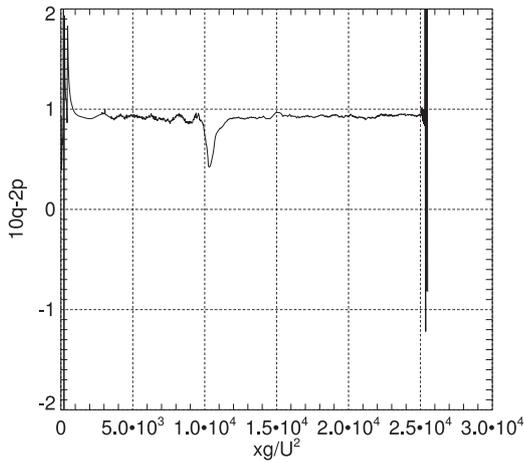


Fig. 22. Relation $(10q - 2p)$ as the function of the fetch for Hsiao-Shemdin wind input term.

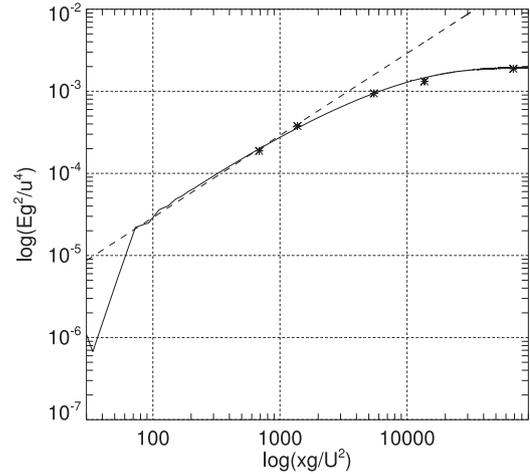


Fig. 24. Decimal logarithm of total dimensionless energy, as a function of decimal logarithm, of the dimensional fetch. Solid line – WAM3 case, dashed line – ZRP case approximated by the fit $2.9 \times 10^{-7} \frac{xg}{U^2}$, stars – data from Fig. 3.7 in monograph (Komen et al., 1994) recalculated in the assumption $u_* = \frac{1}{27} U_{10}$.

than 20 years ago, see the monograph (Komen et al., 1994). The results presented in the current paper do not contradict them (see Fig. 24) and reveal some new features.

The source term for WAM cycles 1 through 3 contain, not only wind input term, but also long-wave dissipation (Komen et al., 1984; Snyder et al., 1981b; Tolman, 2013).

The input source term was used in Snyder form, as per Tolman (2013):

$$S_{in}(k, \theta) = C_{in} \frac{\rho_a}{\rho_w} \max \left[0, \left(\frac{28u_*}{c} \cos(\theta - \theta_w) - 1 \right) \right] \omega \varepsilon(k, \theta) \quad (59)$$

$$u_* = u_{10} \sqrt{(0.8 + 0.065u_{10}) 10^{-3}} \quad (60)$$

where $C_{in} = 0.25$, ρ_a and ρ_w are the densities of air and water, u_* is the wind friction velocity, and c is the wave phase velocity.

White capping dissipation was defined by Eq. (33) (Tolman, 2013). Turning on such dissipation radically changes the whole physical picture of the dissipation-free Snyder case, both quantitatively and qualitatively as well.

First, the dissipation maximum coincides with the spectral maximum. Fig. 23 demonstrating that WAM3 dissipation can be called the “spectral peak dissipation”, indeed due to unambiguous, spectral peak frequency area localization.

Fig. 25a shows that total energy growth, along the fetch, becomes constant, at the dimensionless fetch value $\chi = 5 \times 10^4$, which for wind speed $U = 10$ m/s means 500 km dimensional fetch.

The WAM3 model predicts saturation and formation of the “mature sea”. The limiting level of energy $\varepsilon = 1.9 \times 10^{-3}$ is half that predicted by Pierson–Moscowitz $\varepsilon_{max} \simeq 3.64 \times 10^{-3}$. One should notice that Donelan has predicted $\varepsilon_{max} \simeq 4.07 \times 10^{-3}$, Young has predicted $\varepsilon_{max} \simeq (3.6 \pm 0.9) \times 10^{-3}$. Hence, the WAM3 model essentially underestimates the energy growth, for large values of the fetch. The exact same results are described in the monograph (Komen et al., 1994) (see the corresponding Fig. 3.7).

The novelty is the following – for moderate fetches, $10 < \chi < 10^3$, the WAM3 model gives the same results as ZRP model, *without any spectral dissipation*. This statement is illustrated by Figs. 24 and 25a.

The value of the exponent p versus the fetch, asymptotically, goes to 0, see Fig. 25b. This demonstrates strong discrepancies, with ZRP results, for large fetches.

The stationary level of energy corresponds to 0.2 m^2 , which is approximately 1.5 times less than 0.36 m^2 , the corresponding Pierson–Moscowitz spectrum.

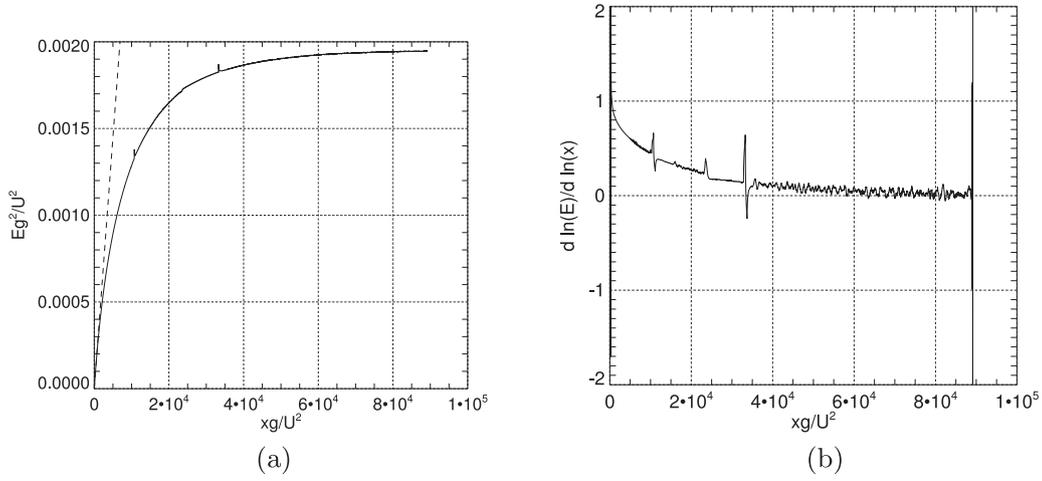


Fig. 25. Same as Fig. 2, but for WAM3 S_{in} .

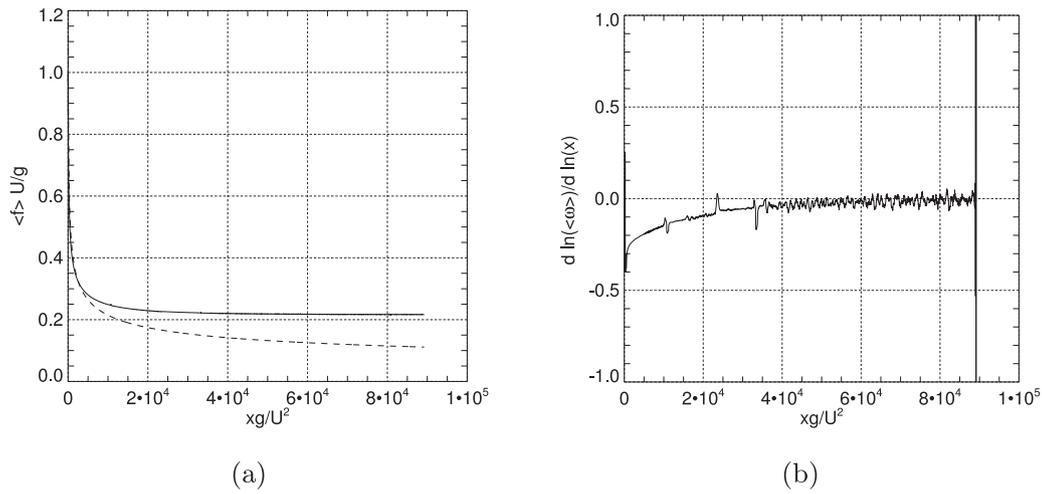


Fig. 26. Same as Fig. 3, but for WAM3 S_{in} .

Similarly to energy, the dependence of the mean frequency, against the fetch, shown in Fig. 26a, becomes constant at the dimensionless fetch value $\chi = 5 \times 10^4$. The value of corresponding index q goes asymptotically to 0, see Fig. 26b. Indicating discrepancies among ZRP results.

Nevertheless, Fig. 27 demonstrates the Kolmogorov–Zakharov segment of the spectrum $\sim \omega^{-4}$, for small fetch value ~ 20 km.

The solid line on Fig. 28 presents, angle averaged, compensated wave energy spectrum, for stationary state, corresponding to dimensional fetch value ~ 500 km (solid line). The dashed line demonstrates compensated Pierson–Moscowitz spectrum. Both spectra have coinciding high-frequency behavior, but are completely different at lower f . Almost perfect correspondence at higher frequencies could mean that WAM3 input terms were tuned to match the experimental results only in spectral tail area.

Fig. 29 presents combination $(10q - 2p)$ as a function of the fetch. It is in total disagreement with the theoretical predictions. There is no indication of “magic relation” Eq. (18) fulfillment.

Comparison of WAM3 model with JONSWAP experiments shows that it describes them fairly well, for small fetches of the order of 10 km, i.e., in the region far from saturation and realization of the stationary state. Applicability of the model, for longer fetches, is questionable, at least for the reason that “magic relation” ceases to be realized.

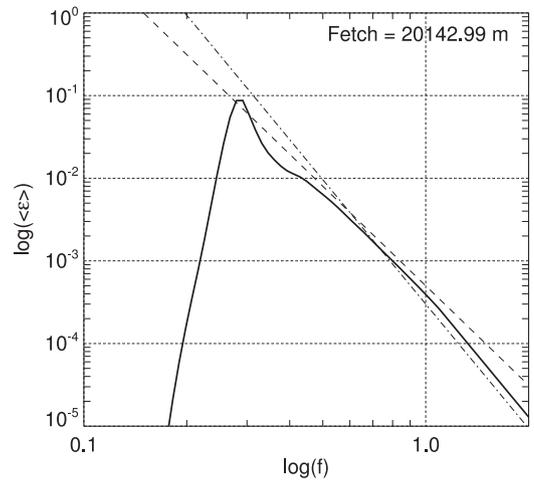


Fig. 27. Same as Fig. 7, but for WAM3 S_{in} .

10. Conclusion

Series of numerical experiments have been performed, for four different variants of wind input terms S_{in} , in the frame of the alternative numerical framework, which assumed:

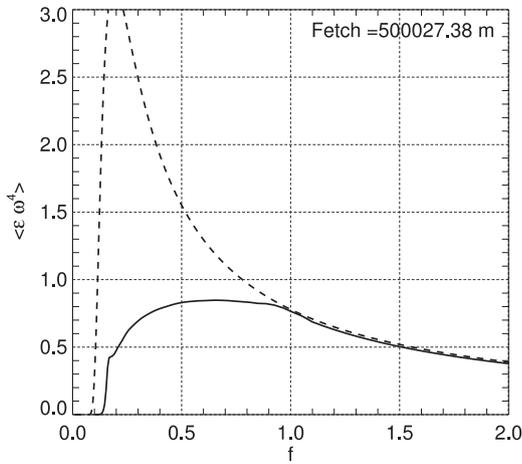


Fig. 28. Angle averaged energy compensated spectrum $\langle \varepsilon \omega^4 \rangle = \frac{1}{2\pi} \int \varepsilon(\omega, \theta) \omega^4 d\theta$ as a function of frequency $f = \frac{\omega}{2\pi}$ for WAM3 S_{in} .

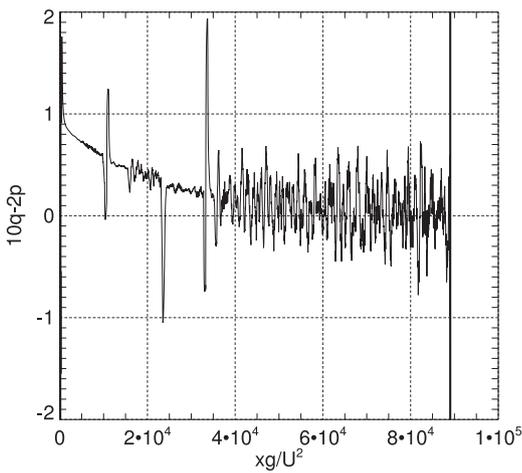


Fig. 29. "Magic relation" $(10q - 2p)$ as a function of the dimensionless fetch for WAM3 wind input term.

1. exact nonlinear term S_{nl}
2. absence of spectral peak dissipation
3. "implicit" high-wavenumbers dissipation in the form of Phillips tail ω^{-5}

The fifth numerical experiment contained WAM3 spectral peak dissipation, "by definition", but all other aspects of the numerical simulation correspond to the above alternative framework.

The results of all five numerical experiments were subjected to the five tests described below, with the summary presented in Table 2.

The p - and q -tests are the checks for Eq. (16) and Eq. (17) respectively; they check if the energy and mean frequency are power functions of the fetch, with proper self-similar exponents $p = 1$ and $q = 0.3$.

The KZ- spectrum test is a direct check of WTT's validity, according to which, the directional (angle averaged) energy spectrum $\langle \varepsilon \omega^4 \rangle$ has to be, with up to 20% accuracy, constant in the inertial interval $1.5\omega_p < \omega < 3.5\omega_p$. Fulfilling this test directly points to the fact that HE, in the universal domain, is described by stationary Eq. (4), which is caused by mutual cancellation of the "in" and "out" terms, as the dominating process in S_{nl} .

The "magic relation" test is a check for the "magic relation" Eq. (18) and is more liberal than the p - and q - tests, since it assumes that power dependencies of the energy and mean frequency, along the fetch, are local, i.e. exponents $p = p(\chi)$ and $q = q(\chi)$ are slow functions of the fetch, but the "magic relation" can still be fulfilled, for any value of the fetch coordinate.

The "energy growth" test is a check that the energy growth rate, versus the fetch, compares with corresponding ZRP dependence and the results of the 12 field experiments.

The following is a discussion of the above tests, applied to five simulations:

1. It is no surprise that the ZRP wind input function passed p - and q - tests, since it was especially designed with the purpose to satisfy them at $p = 1$ and $q = 0.3$. It also passes KZ, "magic relation", and energy growth tests, since it reproduces more than a dozen of the field experiments. Therefore, it can serve as the benchmark.
2. All other wind input terms also pass the KZ- test. Validity of that result, for all five versions of the wind input, suggests its universality. One can say that, if not for all, then for a very wide choice of the wind input functions, the spectrum $\varepsilon \sim \omega^{-4}$ will be realized, due to domination of the conservative terms in the HE.
3. All the cases, except WAM3, passed the "magic relation" test. This means, practically, that for any form of quasi-linear S_{in} , for large fetches, there is formation of a local, self-similar, regime, with indices p and q slowly changing, with the fetch. It also confirms WTT.
4. Chalikov and Hsiao-Shemdin cases fail the p - and q - tests, but are in qualitative agreement with the field experiments.
5. All the cases, except ZRP, fail the energy growth test.

The Table 1, of Section 5, presented the results of 12 experiments, confirming the law of $p = 1$ and $q = 0.3$ indices. Publication (Badulin et al., 2007) presents the data of 24 field experiments - practically everything found in the literature. As it was already mentioned, half of those experiments satisfied p - and q -tests. Simple calculation shows that "magic relation" test is satisfied, with the accuracy of 30%, for 2/3 of the described experiments. The reason for poor performance on the "magic test", by other experiments, is discussed, in detail, in publication (Badulin et al., 2007) and is explained, first of all, by data processing imperfections. More recent experiments, confirming the "magic relation", are presented in publication (Zakharov et al., 2014). One can conclude that the "magic relation" is confirmed, nowadays, experimentally.

As far as the WAM3 model is concerned, while it does not pass any of the tests, except for KZ spectrum test, it realizes speculative phenomenon of the "mature sea", not confirmed by serious

Table 2
Summary of the tests performed on five models of wind input S_{in} .

Experiment	p -test	q -test	KZ-spectrum	Magic relation	Energy growth
ZRP	YES	YES	YES	YES	YES
Chalikov	NO	NO	YES	YES	NO
Snyder	NO	NO	YES	YES	NO
Hsiao--Shemdin	NO	NO	YES	YES	NO
WAM3	NO	NO	YES	NO	NO

experiments, but described in publication (Komen et al., 1984). In any case, none of the field experiments analyzed in Badulin et al. (2007), Zakharov et al. (2014) resemble, even remotely, formation of the “mature sea”. It would seem rational to refrain from this hypothesis.

The main obstacle for self-consistent wind-wave theory creation is the ambiguity of the analytical expression for S_{diss} . Although, it can be resolved, through a numerical solution of, not HE, but primordial Euler equations. Such experiments are already carried out and their result are partially published (Zakharov et al., 2009), (Zakharov et al., 2015). They unequivocally show that long wave dissipation occurs due to wave breaking of the short waves. Long wave dissipation, due to this process, is realized in rogue waves, but they are rare phenomena, and their contribution to energy balance is, at least, orders of magnitude lower than assumed in WAM3 and WAM4 models. This fact also justifies the author’s lack of desire to supply the numerical model, by the long-wave dissipation.

Finally, let’s discuss research perspectives. So far, the authors solved the HE, either as time evolution at a single spatial point (Zakharov et al., 2012), (Pushkarev et al., 2003), (Badulin et al., 2005) (“duration limited setup”), or as a spatially stationary solution (Zakharov et al., 2012) (“fetch limited setup”). Recent progress in the algorithms, computer software, and hardware development allow one to numerically solve the HE, using an exact S_{nl} expression, for a fairly large amount of points in the temporal-spatial domain. Preliminary results on the HE solution, for 40 equidistant points, along the fetch, were already presented at the WISE meeting (Pushkarev and Zakharov, 2013a). Obtained results, for many more points (orders of magnitude larger) will be published soon.

It is expected that the plans to numerically simulate the wind wave development, using exact S_{nl} , are realistic, for fairly small domains like the Black Sea, Lake Michigan, and the Gulf of Bothnia.

The authors believe that the presented research is a step toward radical improvement of the existing operating models. The source functions S_{in} and S_{diss} must be carefully revised and optimized, by numerical experiments using the XNL’s nonlinear term. As soon as this is done, one should choose a proper approximation, for XNL modeling – some version of DIA, with optimally chosen quadruplets. Ideally, during forecasting, the model will not require parameter tuning.

The oceanographic society must start using solid, justified, physical models and abandon the “black box with tuning knobs” approach. A new model will require minimal, if any, tuning for different ocean conditions.

Acknowledgments

The authors would like to thank the referees of the manuscript for valuable discussion, which helped to improve the presented research.

The research presented in Sections 2 and 3 has been accomplished due to support of the grant “Wave turbulence: the theory, mathematical modeling and experiment” of the Russian Science Foundation no 14-22-00174. The research presented in other chapters was supported by ONR grant N00014-10-1-0991. The authors gratefully acknowledge the support of these foundations.

Appendix

It is convenient to present the Hasselmann equation, not for the energy spectrum $\varepsilon(\omega, \theta)$, but for the wave action spectrum $n(k)$. The spectra are connected by the relation

$$\omega_k n_k dk = \varepsilon(\omega, \theta) \omega d\theta \quad (61)$$

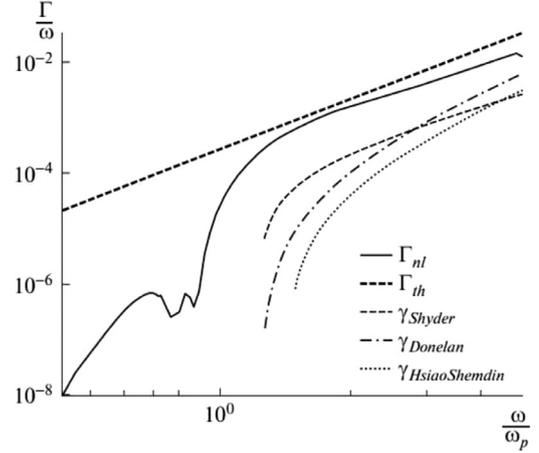


Fig. 30. Coefficient of nonlinear dissipation Γ_k : theoretical estimate(dashed line) (Zakharov, 2010; Zakharov and Badulin, 2011) and numerical calculation (solid line). Known parameterizations, of the wave energy growth rate, are shown on the legend. Essential exceeding of Γ_k over γ_k explains applicability of the equation $S_{nl} = 0$.

The Hasselmann equation reads

$$\frac{\partial n_k}{\partial t} = 2\pi g^2 \int_{k_2, k_3, k_4} (T_{kk_2k_3k_4})^2 \times \quad (62)$$

$$\times (n_k n_{k_3} n_{k_4} + n_{k_2} n_{k_3} n_{k_4} - n_k n_{k_2} n_{k_3} - n_k n_{k_2} n_{k_4}) \times$$

$$\times \delta(\omega + \omega_2 - \omega_3 - \omega_4) \delta(k_1 + k_2 - k_3 - k_4) dk_1 dk_2 dk_3 dk_4$$

$$\text{Here } \omega_k = \sqrt{gk} \text{ and } T_{k_1 k_2 k_3 k_4} = \frac{1}{2} (\tilde{T}_{k_1 k_2 k_3 k_4} + \tilde{T}_{k_2 k_1 k_3 k_4})$$

$$T_{k_1 k_2 k_3 k_4} = \frac{1}{4} \frac{1}{(k_1 k_2 k_3 k_4)^{1/4}} \left\{ \quad (63)$$

$$+ \frac{1}{2} (k_{1+2}^2 - (\omega_1 + \omega_2)^4) [(k_1 \vec{k}_2 - k_1 k_2) + (k_3 \vec{k}_4 - k_3 k_4)]$$

$$- \frac{1}{2} (k_{1-3}^2 - (\omega_1 - \omega_3)^4) [(k_1 \vec{k}_3 - k_1 k_3) + (k_2 \vec{k}_4 + k_2 k_4)]$$

$$- \frac{1}{2} (k_{1-4}^2 - (\omega_1 - \omega_4)^4) [(k_1 \vec{k}_4 + k_1 k_4) + (k_2 \vec{k}_3 + k_2 k_3)]$$

$$+ \left(\frac{4(\omega_1 + \omega_2)^2}{k_{1+2} - (\omega_1 + \omega_2)^2} - 1 \right) (k_1 \vec{k}_2 - k_1 k_2) (k_3 \vec{k}_4 + k_3 k_4)$$

$$+ \left(\frac{4(\omega_1 - \omega_3)^2}{k_{1-3} - (\omega_1 - \omega_3)^2} - 1 \right) (k_1 \vec{k}_3 + k_1 k_3) (k_2 \vec{k}_4 + k_2 k_4)$$

$$+ \left(\frac{4(\omega_1 - \omega_4)^2}{k_{1-4} - (\omega_1 - \omega_4)^2} - 1 \right) (k_1 \vec{k}_4 + k_1 k_4) (k_2 \vec{k}_3 + k_2 k_3) \left. \right\}$$

where notation $k_{1+2} = |\vec{k}_1 + \vec{k}_2|$.

Eq. (63) can be rewritten as follows:

$$\frac{\partial n_k}{\partial t} = S_{nl} = F_k - \Gamma_k n_k \quad (64)$$

where

$$F_k = 2\pi g^2 \int |T_{kk_1 k_2 k_3}|^2 n_{k_1} n_{k_2} n_{k_3} \times \delta(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) dk_1 dk_2 dk_3 \quad (65)$$

$$\Gamma_k = 2\pi g^2 \int |T_{kk_1 k_2 k_3}|^2 (n_{k_1} n_{k_2} + n_{k_1} n_{k_3} - n_{k_2} n_{k_3}) \times \quad (66)$$

$$\times \delta(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) dk_1 dk_2 dk_3$$

KZ stationary spectra are given by the expression

$$n_k = \frac{F_k}{\Gamma_k} \quad (67)$$

As far as $n_k > 0$, $\Gamma_k > 0$. The stationary kinetic equation, in the presence of wind input and damping, reads

$$S_{nl} + \gamma_k n_k = 0 \quad (68)$$

Here $\gamma_k = \gamma_{in} - \gamma_{diss}$. The solution to this equation is

$$n_k = \frac{F_k}{\Gamma_k - \gamma_k} \quad (69)$$

As far as $n_k > 0$, then $\Gamma_k > \gamma_k$. In fact, in the real situation $\Gamma_k > \gamma_k$. This is clear from Fig. 30, adopted from paper (Zakharov and Badulin, 2011).

It should be stressed out that our modernized code made possible separate calculation of F_k and Γ_k .

Here the dashed line is the theoretical calculation of Γ_k , made narrow in the angle spectrum, and the solid line is the numerical experiment. More details can be found in paper (Zakharov, 2010).

Excess of Γ_k over γ_k , by orders of magnitude, explains the dominant role of S_{nl} .

References

- Badulin, S.I., Babanin, A.V., Resio, D., Zakharov, V.E., 2007. Weakly turbulent laws of wind-wave growth. *JFM* 591, 339–378.
- Badulin, S.I., Pushkarev, A.N., D.Resio, Zakharov, V.E., 2005. Self-similarity of wind-driven sea. *Nonlinear Proc. Geophys.* 12, 891–945.
- Badulin, S. I., Zakharov, V. E., 2012. The generalized Phillips' spectra and new dissipation function for wind-driven seas. arXiv:1212.0963[physics.ao-ph], 1–16.
- Banner, M.L., X.Tian, 1998. On the determination of the onset of breaking for modulating surface gravity water waves. *J. Fluid Mech.* 367, 107–137.
- Belcher, S.E., Hunt, J.C.R., 1998. Turbulent flow over hills and waves. *Annu. Rev. Fluid Mech.* 30, 507–538.
- Bonmarin, P., 1989. Geometric properties of deep-water waves. *J. Fluid Mech.* 209, 405–433.
- C. Connaughton, Y.P., 2004. Kinetic theory and Bose Einstein condensation. *C. R. Phys.* 5, 015301.
- Chalikov, D., 1995. The parameterization of the wave boundary layer. *JPO* 25, 1333–1349.
- Chalikov, D.V., Makin, V.K., 1991. Models of the wave boundary layer. *Boundary Layer Meteorol.* 56, 83–99.
- Crapper, G.D., 1957. An exact solution for progressive capillary waves of arbitrary amplitude. *J. Fluid Mech.* 2, 532–540.
- Donelan, M., Babanin, A.V., Young, I.R., Banner, M., 2006. Waves-follower field measurements of the wind-input spectral function. Part II: parameterization of the wind input. *JPO* 36, 1672–1689.
- Donelan, M., Babanin, A.V., Young, I.R., Banner, M.L., McCormick, C., 2005. Wave-follower field measurements of the wind-input spectral function. Part I: measurements and calibrations. *J. Atmos. Oceanic Technol.* 22, 799–813.
- Donelan, M.A., Hamilton, J., Hui, W.H., 1985. Directional spectra of win-generated waves. *Philos. Trans. R. Soc. Lond. A* 315, 509–562.
- Dyachenko, A.I., I.Kachulin, D., Zakharov, V.E., 2015. Evolution of one-dimensional wind-driven sea spectra. *JETP Lett.* 102, 577–581.
- Dyachenko, A.I., Zakharov, V.E., 1994. Is free-surface hydrodynamics an integrable system? *Phys. Lett. A* 190, 144–148.
- Dyachenko, A.I., Zakharov, V.E., 2005. Modulation instability of stokes wave-free wave. *JETP Lett.* 81, 255–259.
- Fabricant, A.L., 1976. Quasilinear theory of wind-waves generation. *Izv. Atmos. Ocean. Phys.* 12, 858–862.
- Gagnaire-Renou, E., Benoit, M., Badulin, S., 2011. On weakly turbulent scaling of wind sea in simulation of fetch-limited growth. *JFM* 669, 178–213.
- Galtier, A.C.A.P.S., Nazarenko, S.V., 2000. A weak turbulence theory for incompressible MHD. *J. Plasma Phys.* 63, 447.
- Galtier, S., 2003. Weak inertial-wave turbulence theory. *Phys. Rev. E* 68, 015301.
- Gemmrich, J., Garrett, C., 2008. Unexpected waves. *JPO* 38, 2330–2336.
- Gemmrich, J.R., Banner, M.L., Garrett, C., 2008. Spectrally resolved energy dissipation rate and momentum flux of breaking waves. *JPO* 38, 1296–1312.
- Geogjaev, V.V., Zakharov, V.E., 2015. On interaction of short and long waves on the surface of liquid. *JETP Lett.* 10, 509–537.
- Hasselmann, D., Bosenberg, J., 1991. Field measurements of wave-induced pressure over wind-sea and swell. *JFM* 230, 391–428.
- Hasselmann, K., 1962. On the non-linear energy transfer in a gravity-wave spectrum. Part 1. General theory. *J. Fluid Mech.* 12, 481–500.
- Hasselmann, K., 1963. On the non-linear energy transfer in a gravity wave spectrum. Part 2. Conservation theorems; wave-particle analogy; irreversibility. *J. Fluid Mech.* 15, 273–281.
- Hristov, T.S., Miller, S.D., Friehe, C.A., 2003. Dynamical coupling of wind and ocean waves through wave-induced air flow. *Nature* 422, 55–58.
- Hsiao, S.V., Shemdin, O.H., 1983. Measurements of wind velocity and pressure with a wave follower during MARSEN. *JGR* 88, 9841–9849.
- Huang, P., Toporkov, Y., Sitten, M., Menk, S., 2013. Mapping surface currents and waves with interferometric synthetic aperture radar in coastal waters: observation of wave breaking in swell-dominant conditions. *JPO* 43, 563–581.
- Hwang, P., Toporkov, Y., Sitten, M., Menk, S., 2013. Measuring wave breaking by radar. In: WISE Meeting, College Park, MD, USA.
- Hwang, P.A., 2007. Spectral signature of wave breaking in surface wave components of intermediate-length scale. *J. Mar. Syst.* 66, 28–37.
- Hwang, P.A., Wang, D.W., 2004. An empirical investigation of source term balance of small scale surface waves. *Geophys. Res. Lett.* 31, L15301.
- Janssen, P., 2009. The Interaction of Ocean Waves and Wind. Cambridge Monographs on Mechanics and Applied Mathematics. Cambridge University Press.
- Janssen, P.A.E.M., 1982. Quasilinear approximation for the spectrum of wind-generated water waves. *JFM* 117, 493–506.
- Jeffreys, H., 1924. On the formation of water waves by wind. *Proc. Roy. Soc.* 107A, 189–206.
- Jeffreys, H., 1925. On the formation of water waves by wind II. *Proc. Roy. Soc.* 107A, 341–347.
- Katz, A.V., Kontorovich, V.M., 1975. Power law solution of Boltzmann kinetic equation, describing the spectral distribution of particles with fluxes. *Pis. Zh. Exp. Teor. Fiz.* 21, 13.
- Kleiss, J.M., Melville, W.K., 2002. Observation of wave breaking kinematics in fetch-limited seas. *JPO* 40, 2575–2604.
- Komatsu, K., Masuda, A., 1996. A new scheme of nonlinear energy transfer among wind waves: RIAM method. algorithm and performance. *J. Oceanogr.* 52, 509–537.
- Komen, G.J., Cavaleri, L., Donelan, M., Hasselmann, K., Hasselmann, S., Janssen, P.A.E., 1994. Dynamics and Modeling of Ocean Waves. Cambridge University Press.
- Komen, G.J., Hasselmann, S., Hasselmann, K., 1984. On the existence of a fully developed wind-sea spectrum. *J. Phys. Oceanogr.* 14, 1271–1285.
- Korotkevich, A.O., Pushkarev, A.N., Resio, D., Zakharov, V.E., 2008. Numerical verification of the weak turbulent model for swell evolution. *Eur. J. Mech. B—Fluids* 27, 361–387.
- Kudryavtsev, V.N., Makin, V.K., Meirink, J.F., 2001. Simplified model of the air flow above the waves. *Boundary Layer Meteorol.* 98, 155–171.
- Landau, L.D., Lifshitz, E.M., 1980. Statistical Physics: Theory of the Condensed State. Butterworth-Heinemann.
- Lavrenov, I.V., 2010. Wind-waves in Oceans: Dynamics and Numerical Simulations. Springer.
- Long, C., Resio, D., 2007. Wind wave spectral observations in Currituck Sound, North Carolina. *JGR* 112, C05001.
- Longuet-Higgins, M.S., 1980a. On the forming of sharp corners at a free surface. *Proc. R. Soc. Lond. A* 371, 453–478.
- Longuet-Higgins, M.S., 1980b. A technique for time-dependent, free-surface flow. *Proc. R. Soc. Lond. A* 371, 441–451.
- Longuet-Higgins, M.S., 1981. On the overturning of gravity waves. *Proc. R. Soc. Lond. A* 376, 377–400.
- Longuet-Higgins, M.S., 1982. Parametric solutions for breaking waves. *J. Fluid Mech.* 121, 403–424.
- Lvov, Y.V., Newell, A.C., 2000. Finite flux solutions of the quantum Boltzmann equation and semiconductor lasers. *Phys. Rev. Lett.* 84, 1894.
- Mastenbroek, C., Makin, V.K., Garat, M.H., Giovanangeli, J.P., 1996. Experimental evidence of the rapid distortion of turbulence in the air flow over water waves. *JFM* 318, 273–302.
- Miles, J., 1957. On the generation of surface waves by shear flows. *JFM* 3, 185–204.
- Newell, A.C., Zakharov, V.E., 2008. The role of the generalized Phillips' spectrum in wave turbulence. *Phys. Lett. A* 372, 4230–4233.
- Nikolaeva, Y.L., Zymring, L.S., 1986. Kinetic model of sea wind waves generation by turbulent wind. *Izv. Atmos. Ocean. Phys.* 22, 135–142.
- Nordheim, L.W., 1928. On the kinetic method in the new statistics and its application in the electron theory of conductivity. *Proc. R. Soc. Lond. A* 119, 689–698.
- Perrie, W., Zakharov, V.E., 1999. The equilibrium range cascades of wind-generated waves. *Eur. J. Mech. B/Fluids* 18, 365–371.
- Phillips, O.M., 1966. The Dynamics of the Upper Ocean. Cambridge Monographs on Mechanics and Applied Mathematics. Cambridge University Press.
- Phillips, O.M., 1985. Spectral and statistical properties of the equilibrium range in wind-generated gravity waves. *JFM* 156, 505–531.
- Plant, W.J., 1982. A relationship between wind stress and wave slope. *JGR* 87, 1961–1967.
- Pushkarev, A., Resio, D., Zakharov, V., 2003. Weak turbulent approach to the wind-generated gravity sea waves. *Physica D* 184, 29–63.
- Pushkarev, A., Zakharov, V.E., 2013. Nonlinear generation of surface waves against the wind in a limited fetch growth model. In: WISE Meeting, College Park, MD, USA.
- Pushkarev, A., Zakharov, V.E., 2013. Quasibreathers in the MMT model. *Physica D* 248, 55–61.
- Resio, D., Perrie, W., 1991. A numerical study of nonlinear energy fluxes due to wave-wave interactions. Part I: methodology and basic results. *J. Fluid Mech.* 223, 603.
- Resio, D., Perrie, W., 1989. Implications of an f^{-4} equilibrium range for wind-generated waves. *JPO* 19, 193–204.
- Resio, D.T., Long, C.E., Vincent, C.L., 2004. Equilibrium-range constant in wind-generated wave spectra. *JGR* 109, C01018.
- Romero, L., Melville, W.K., 2010. Airborne observations of fetch-limited waves in the gulf of Tehuantepec. *JPO* 40, 441–458.
- Snyder, R.L., Dobson, F.W., Elliott, J.A., Long, R.B., 1981. Array measurements of atmospheric pressure fluctuations above surface gravity waves. *JFM* 102, 1–59.
- Snyder, R.L., Dobson, F.W., Elliott, J.A., Long, R.B., 1981. Array measurements of atmospheric pressure fluctuations above surface gravity waves. *J. Fluid Mech.* 102, 1–59.

- Song, J.B., Banner, M.L., 2002. On determining the onset and strength of breaking for deep water waves. Part I: unforced irrotational wave groups. *JPO* 32, 2541–2558.
- Sverdrup, H.U., Munk, W.H., 1947. *Wind, Sea and Swell: Theory of Relations for Forecasting*. US Hydrographic Office, vol. H.O. Pub. No. 601.
- Tolman, H.L., 2013. User Manual and System Documentation of WAVEWATCH III. Environmental Modeling Center, Marine Modeling and Analysis Branch.
- Tolman, H.L., Chalikov, D., 1996. Source terms in a third-generation wind-wave model. *JPO* 26, 2497–2518.
- Tracy, B., Resio, D., 1982. Theory and Calculation of the Nonlinear Energy Transfer between Sea Waves in Deep Water. WES report 11.
- Troitskaya, Y., Sergeev, D., Ermakova, O., Balandina, G., 2011. Statistical parameters of the air boundary layer over steep water waves measured by PIV technique. *JPO* 41, 1421–1454.
- Tsagareli, K., Babanin, A., Walker, D., Young, I., 2009. Numerical investigation of spectral evolution of wind waves. *JPO* 40, 656–666.
- Vledder, G.P.V., 2006. The WRT method for the computation of nonlinear four wave interactions in discrete spectral wave models. *Coastal Engineering* 53, 223–242.
- Webb, D.J., 1978. Non-linear transfers between sea waves. *Deep Sea Res.* 25, 279–298.
- Young, I.R., 1999. *Wind Generated Ocean Waves*. Elsevier.
- Young, I.R., Babanin, A.V., 2006. Spectral distribution of energy dissipation due to dominant wave breaking. *JPO* 36, 376–394.
- Zakharov, V.E., 2005. Theoretical interpretation of fetch-limited wind-driven sea observations. *NPG* 13, 1–16.
- Zakharov, V.E., 2010. Energy balances in a wind-driven sea. *Phys. Scr.* T142, 014052.
- Zakharov, V. E., 2015. Direct and inverse cascade in the wind-driven sea. arXiv:1502.02628[physics.ao-ph].
- Zakharov, V.E., 2015. Integrability of free surface hydrodynamics for unveiling flow. *J. Fluid Mech.* in press.
- Zakharov, V.E., Badulin, S.I., 2011. On energy balance in wind-driven sea. *Dokl. Akad. Nauk.* 440, 691–695.
- Zakharov, V. E., Badulin, S. I., Hwang, P. A., Caulliez, G., 2014. Universality of sea wave growth and its physical roots. arXiv:1411.7235[physics.ao-ph], 34, *JFM*, in press.
- Zakharov, V.E., Dyachenko, A.I., 2008. On the formation of freak waves on the surface of deep water. *JETP Lett.* 88, 307–311.
- Zakharov, V. E., Dyachenko, A. I., 2012. Free-surface hydrodynamics in the conformal variables. arXiv:1206.2046.
- Zakharov, V.E., Filonenko, N.N., 1967. The energy spectrum for stochastic oscillations of a fluid surface. *Sov. Phys. Dokl.* 11, 881–884.
- Zakharov, V.E., Kachulin, D.I., Dyachenko, A.I., 2015. On dissipation of ocean spectra due to wave-breaking. *Dokl. RAS* 102. In press.
- Zakharov, V.E., Korotkevich, A.O., Prokofiev, A.O., 2009. On dissipation function of ocean waves due to whitecapping. In: Simos, T.E., Psihoyios, G., Tsitouras, C. (Eds.), *American Institute of Physics Conference Series*, Vol. 1168, pp. 1229–1237.
- Zakharov, V.E., Kuznetsov, E.A., 2012. Solitons and collapses—two scenarios of the evolution of nonlinear wave systems. *Phys. Uspekhi* 55, 535–556.
- Zakharov, V.E., L'vov, V.S., Falkovich, G., 1992. *Kolmogorov Spectra of Turbulence I: Wave Turbulence*. Springer-Verlag.
- Zakharov, V.E., Pushkarev, A.N., 1999. Diffusion model of interacting gravity waves on the surface of deep fluid. *NPG* 6, 1–10.
- Zakharov, V. E., Resio, D., Pushkarev, A., 2012. New wind input term consistent with experimental, theoretical and numerical considerations. arXiv:1212.1069[physics.ao-ph], *JPO*.